Math 300 Section D
Summer 2014
Final
July 26, 2014
Time Limit: 60 Minutes

Name (Print): ____________________

Student ID: ____________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

1. (10 points) For each of the following, give an example of a relation satisfying the specified properties. Prove that it satisfies these properties
   (a) (5 points) a relation that is reflexive and symmetric but not transitive
   (b) (5 points) a relation that is symmetric and transitive but not reflexive

2. (10 points) Let A, B be sets. Prove that

$$\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$$

3. (10 points) Let A and B be sets. State the definition of each of the following phrases. (Note: when stating definitions, precision counts!)

   (a) (2 points) a *relation from* A to B

   (b) (2 points) a *function* from A to B

   (c) (2 points) an *equivalence relation* on A

   (d) (2 points) a *injective function* from A to B

   (e) (2 points) the sets A and B are *equinumerous*

4. (10 points) Use induction to show that for all integers $n > 0$,

$$8|(5^n + 12n - 1).$$

5. (10 points) **True or False Section:**

- For each of the following determine if the statement is true or false.
- If it is true, write TRUE. If it is false write FALSE.
- If you write T or F, or anything else it will be considered a non-response.

(a) (2 points) f^{-1} is a function if and only if f is injective

(b) (2 points) a set A and its power set $\mathcal{P}(A)$ are never equinumerous

(c) (2 points) if A and B are finite sets, then a function $f : A \to B$ is injective if and only if it is surjective

(d) (2 points) if R is a total ordering of a set A, then every subset of A has an R-smallest element

(e) (2 points) if R is an equivalence relation on a set A and $a \in A$, then $x \in [a]_R \Rightarrow [x]_R = [a]_R$

6. (10 points) Let A, B, C be sets, and $f : A \to B$ and $g : B \to C$. Consider each of the following statements. If the statement is true, prove it. If the statement is false, provide a counter-example.

   (a) (5 points) If $g \circ f$ is one-to-one, then so too is g

   (b) (5 points) If $g \circ f$ is one-to-one, then so too is f