Math 300 Section D
Summer 2014
Exam 1
July 26, 2014
Time Limit: 60 Minutes

Name (Print): ____________________

Student ID: ____________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

Do not write in the table to the right.

1. (10 points) In each of the following questions, show that the two statements are equivalent by finding a sequence of equivalent expressions connecting the two. Each equivalence must be justified by an appropriate identity. (No credit for truth table)

   (a) (5 points) $(P \vee Q) \vee \neg(\neg P \vee \neg R)$ and $P \vee Q$

   (b) (5 points) $(P \vee \neg(\neg P \wedge \neg Q)) \wedge \neg((\neg P \wedge R) \vee (R \wedge \neg R))$ and $P \vee (Q \wedge \neg R)$

2. (10 points) Let A, B, C be sets. Prove each of the following identites

   (a) (5 points) $A \cap (B \cup (A \cap B)) = A \cap B$

   (b) (5 points) $(A \cap B) \backslash (B \cap C) = (A \cap B) \backslash C$

3. (10 points) In each of the following questions, show that the two statements are equivalent by finding a sequence of equivalent expressions connecting the two. Each equivalence must be justified by an appropriate identity. (No credit for truth table)

   (a) (5 points) $P \Leftrightarrow (Q \Rightarrow (P \vee R))$ and $P \vee (Q \wedge \neg R)$

   (b) (5 points) $(P \wedge \neg Q) \Rightarrow (P \Rightarrow Q)$ and $\neg P \vee Q$

4. (10 points) Let $\mathcal{F}$ and $\mathcal{G}$ be families of sets. Prove that

$$\cup(\mathcal{F} \cap \mathcal{G}) \subseteq (\cup\mathcal{F}) \cap (\cup\mathcal{G}).$$

5. (10 points) For each of the following statements, either prove the statement or provide a counter-example.

   (a) (5 points) Let n be an integer. Then n is divisible by 12 if and only if n is divisible by 4 and n is divisible by 3.

   (b) (5 points) Let n be an integer. Then n is divisible by 18 if and only if n is divisible by 3 and n is divisible by 10.

6. (10 points) Let n be an integer. Prove that x^4 is divisible by 8 or $x^4 - 1$ is divisible by 8.