MATH 300: Problem Set #5

Due on: August 15, 2014

Problem 1 *Velleman Problems*

- pp. 233-236 # 4,8,9a,14,15
- pp. 243-245 # 1,4,6,8,15
- pp. 253-255 # 4,6,11,12

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Problem 2 *The Pidgeonhole Principal*

For this problem, we will need to use the Pidgeonhole Principal. This charmingly named concept is a mathematical formalization of a very intuitive concept: if you are trying put $n+1$ objects in n containers, then at least one container will have to hold more than one object. A mathematically formal way of thinking about this is the following theorem.

Theorem 1 (The Pidgeonhole Principal)**.** Let A and B be finite sets and $f : A \to B$. If the number of elements in A is greater than the number of elements of B, then f is not one-to-one.

After we have covered induction, we will be able to prove this theorem. Until then, we must be content to use it without proof. Now solve the following problem.

Let A be a set consisting of exactly ten distinct integers between 1 and 100. Also let B be the set of all positive integers between 0 and 1000. Consider the function $f : \mathcal{P}(A) \to B$ defined by the rule that for all $X \in \mathcal{P}(A)$,

$$f(X) = \text{sum of the elements of } X.$$

(a) Prove that f is not one-to-one.

(b) Using (a), prove that there exist subsets C, D of A with $C \neq D$ such that the sum of the elements in C is equal to the sum of the elements in D

(c) Using (b), prove that there exist *disjoint* non-empty subsets C', D' of A with the sum of the elements in C' equal to the sum of the elements in D'

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Problem 3 *Another Pidgeonhole Problem*

You find a beat-up old chess board that is missing the A1 and H8 squares (they have been torn off, but all other squares are present). As a super-fancy art project, you decide to glue dominoes to the surface of the board. Assuming each domino takes up exactly two adjacent squares, is it possible to cover the entire chess board with dominoes? (Hint: think about white vs. black squares)

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