Math 307 Section F
Spring 2013
Exam 2
May 22, 2013
Time Limit: 50 Minutes

Name (Print): _____

Student ID: _____

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a single, handwritten, one-sided notesheet and a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

Do not write in the table to the right.

1. (10 points) Solve the following initial value problem

$$y'' + 6y' + 9y = 0, \quad y(0) = 1, \; y'(0) = 0$$

2. **Propose a Solution Section!**

Directions: The "Propose a Solution" section consists of five linear nonhomogeneous equations. For each of these equations, write down the type of function y (with undetermined coefficients) you would try, in order to get a particular solution. *You do NOT need to solve the equations* For example, if the equation were

$$y'' + 2y' + y = e^t,$$

a *correct answer* would be

$$y = Ae^t,$$

and *incorrect answers* would include

$$y = (At + B)e^t, \quad y = At^2e^{2t}, \quad y = Ae^{3t}, \quad y = A\pi^t$$

Each part is worth 2pts:

(a) (2 points)

$$y'' + 2y' + 2y = t^3e^{4t}$$

(b) (2 points)

$$3y'' + 6y' + 3y = (t + 1)e^{-t}$$

(c) (2 points)

$$y'' + 4y' - 5y = e^t$$

(d) (2 points)

$$y'' + 2y' = t^2 + 1$$

(e) (2 points)

$$y'' - 3y' + 2y = e^{2t}$$

3. (10 points)

(a) (5 points) Show that the equation

$$y\cos(xy) + \cos(x) + (x\cos(xy) + 3y^2)y' = 0.$$

is exact. Then solve it.

(b) (5 points) Find an integrating factor for the equation

$$2xe^{x^2} + 3y^2(e^{x^2} + 1)y' = 0$$

You do not need to solve it.

4. (10 points)

(a) (4 points) Find a particular solution to the equation

$$y'' + 2y' + y = \cos(t)e^t$$

(b) (2 points) Find a particular solution to the equation

$$y'' + 2y' + y = \sin(t)e^t$$

(c) (2 points) Find a particular solution to the equation

$$y'' + 2y' + y = 4\cos(t)e^t - 7\sin(t)e^t$$

(d) (2 points) Write down the general solution to the equation

$$y'' + 2y' + y = 4\cos(t)e^t - 7\sin(t)e^t$$

5. (10 points) Given that $y_1 = e^t$ is a solution to the differential equation

$$ty'' - (t+1)y' + y = 0,$$

use the method of reduction of order to find the general solution of the equation.

6. (10 points) Suppose a certain spring is known to respond to a force of 2 lbs by stretching 6 inches. We attach a mass m to this spring and submerge it in a liquid. Experimentally, it is known that the liquid exerts a damping force on the mass spring system, with damping constant $\gamma = 2$ lbs·s/ft.

   (a) For what values of m is the mass spring system overdamped?

   (b) Suppose that the mass we attach weighs 8 lbs, and that the system is initially contracted 6 inches from its equilibrium and then released. Find the position u of the mass (relative to it's equilibrium position) as a function of time.