MATH 307: Problem Set #1

Due on: Oct 3, 2014

Problem 1 *First Order Linear Equations*

For each of the following equations:

(i) Use a computer to graph the slope field of the differential equation.
Include a printout of your graph with your homework

(ii) Based on inspection of the direction field, describe how you expect solutions to behave for large values of t

(iii) Find the general solution to the equation and use it to determine how the solution behaves as $t \to \infty$.

(a) $y' - 2y = t^2 e^{2t}$

(b) $ty' + 2y = \sin(t)$, for $t > 0$

(c) $ty' - y = t^2 e^{-t}$, for $t > 0$

. . . .

Problem 2 *First Order Linear Initial Value Problems*

Use the method of integrating factors to find a solution to each of the given initial value problems

(a) $y' + 2y = te^{-2t}$, $y(1) = 0$

(b) $y' + 2y/t = \frac{1}{t^2}\cos(t)$, $y(\pi) = 0$

(c) $ty' + 2y = \sin(t)$, $y(\pi/2) = 1$

(d) $t^3 y' + 4t^2 y = e^{-t}$, $y(-1) = 0$

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Problem 3 *Variation of Parameters*

Use variation of parameters to find the general solution of the given differential equation

(a) $y' + y/t = 3\cos(2t)$

(b) $2y' + y = 3t^2$

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Problem 4 *Separable Equations*

In each of the following, find a family of solutions parametrized by a constant

(a) $y' = \frac{x^2}{y(1+x^3)}$

(b) $y' + y^2 \sin(x) = 0$

(c) $\frac{dy}{dx} = \frac{x-e^{-x}}{y+e^y}$

(d) $\frac{dy}{dx} = \frac{x^2}{1+y^2}$

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Problem 5 *Separable Initial Value Problems*

For each of the following initial value problems

(i) Solve the initial value problem

(ii) Using a computer, graph the solution
Attach a printout of your graph to your homework

(iii) Determine as accurately as you can the interval in which the solution is defined

(a) $xdx + ye^{-x}dy = 0$, $y(0) = 1$

(b) $y' = \frac{3x^2-e^x}{2y-5}$, $y(0) = 1$

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Problem 6 *Homogeneous Equations*

For each of the following, show that the equation is homogeneous. Then find a family of solutions differing by a constant

(a) $\frac{dy}{dx} = \frac{3y^2-x^2}{2xy}$

(b) $\frac{dy}{dx} = -\frac{4x+3y}{2x+y}$

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