Math 307 Quiz 1

September 26, 2014

Problem 1. Find a solution to the following initial value problem

$$(1/t)y' = ye^t, \quad y(0) = 1.$$

Solution 1. We first separate:

$$(1/y)y' = te^t$$

Then we “multiply” both sides by dx and integrate:

$$\int (1/y)dy = \int te^t dt.$$

Actually performing these integrals, we obtain

$$\ln|y| = te^t - e^t + C_0.$$

Then solving for y:

$$y = C_1 e^{te^t - e^t},$$

where $C_1 = e^{C_0}$. Then since $y(0) = 1$, we know that $1 = C_1/e$, and so our solution is

$$y = e^{te^t - e^t + 1}.$$

Problem 2. Find a solution to the following initial value problem

$$y' = x\cos^2(y/x) + y/x, \quad y(1) = 0.$$

Solution 2. This equation is homogeneous! So we first substitute

$$z = y/x$$

and

$$y' = z + xz',$$

and our equation becomes

$$z + xz' = x\cos^2(z) + z.$$

Simplifying this, we find

$$z' = \cos^2(z),$$

which is separable. We separate:

$$\sec^2(z)z' = 1$$

and then multiply both sides by dx and integrate

$$\int \sec^2(z)dz = \int 1dx.$$

Then actually doing the integral, we find

$$\tan(z) = x + C,$$

so that $z = \tan^{-1}(x + C)$ and therefore

$$y = xz = x\tan^{-1}(x + C).$$

Then since $y(1) = 0$, we find $0 = \tan^{-1}(C + 1)$ and therefore $C + 1 = 0$, so that $C = -1$. Thus our solution is

$$y = x\tan^{-1}(x - 1).$$