MATH 307: Problem Set #3

Due on: October 20, 2014

Problem 1 *Autonomous Equations*

Recall that an equilibrium solution of an autonomous equation is called *stable* if solutions lying on both sides of it tend toward it; is called *unstable* if solutions lying on both sides tend away from it; and is called *semistable* if solutions lying on one side of it tend toward it, while solutions on the other side tend away. In Prob 2.i-2.vi, please do each of the following

(a) Sketch a graph of $f(y)$ versus y

(b) Determine the critical (equilibrium) points

(c) Classify the critical points as stable, semistable, or unstable

(d) Draw the phase line and sketch several graphs of the solution in the ty-plane.

(i) $dy/dt = ay + by^2$, where $a > 0$, $b > 0$ and $y_0 \geq 0$

(ii) $dy/dt = ay + by^2$, where $a > 0$, $b > 0$ and $-\infty < y_0 < \infty$

(iii) $dy/dt = e^y - 1$, $-\infty < y_0 < \infty$

(iv) $dy/dt = e^{-y} - 1$, $-\infty < y_0 < \infty$

(v) $dy/dt = y(1 - y^2)$, $-\infty < y_0 < \infty$

(vi) $dy/dt = y^2(4 - y^2)$, $-\infty < y_0 < \infty$

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Problem 2 *Euler's Method*

For Prob 1.i and 1.ii please do each of the following

(a) Find approximate values of the solution of the given value problem in the interval $[0, 0.5]$ with $\Delta t = 0.100$ using Euler's method. Record your results as a table of values in your writeup.

(b) Find approximate values of the solution of the given value problem in the interval $[0, 0.5]$ with $\Delta t = 0.050$ using Euler's method. Record your results as a table of values in your writeup.

(c) Find approximate values of the solution of the given value problem in the interval $[0, 0.5]$ with $\Delta t = 0.025$ using Euler's method. Record your results as a table of values in your writeup.

(d) Find the exact solution to the initial value problem.

(e) Compare your results in parts (a), (b), (c), and (d) by plotting them all in the same graph. Be sure that your plot is clear enough that one can tell which curves correspond to each part.

(i) $y' = 2y - 1$, $y(0) = 1$

(ii) $y' = 3\cos(t) - 2y$, $y(0) = 0$

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Problem 3 *Exact Equations*

In each of the following, determine if the equation is exact. If it is exact, then find the solution.

(i) $(2x + 4y) + (2x - 2y)y' = 0$

(ii) $(2xy^2 + 2y) + (2x^2y + 2x)y' = 0$

(iii) $\frac{dy}{dx} = -\frac{ax-by}{bx-cy}$

(iv) $(e^x \sin y + 3y)dx - (3x - e^x \sin y)dy = 0$

(v) $(y/x + 6x)dx + (\ln(x) - 2)dy = 0$

(vi) $\frac{xdx}{(x^2+y^2)^{3/2}} + \frac{ydy}{(x^2+y^2)^{3/2}} = 0$

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Problem 4 *Integrating Factors*

For each of the following, find an integrating factor and solve the given equation

(i) $y' = e^{2x} + y - 1$

(ii) $ydx + (2xy - e^{-2y})dy = 0$

(iii) $\left(3y + \frac{\sin(y)}{x^2y}\right) dx + \left(2x + \frac{\cos(y)}{xy}\right) dy = 0$ [Hint: Try $\mu(x, y) = x^a y^b$]

(iv) $\left(3x + \frac{6}{y}\right) + \left(\frac{x^2}{y} + 3\frac{y}{x}\right) \frac{dy}{dx} = 0$ [Hint: Try $\mu(x, y) = \mu(xy)$]

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