Math 307 Quiz 3

October 20, 2014

Problem 1. Use the method of variation of parameters to find the *general* solution to the differential equation

$$y' + y = \sin(t).$$

Solution 1. We first solve the corresponding homogeneous equation

$$y_h' + y_h = 0.$$

This equation is separable, and a solution is $y_h = e^{-t}$. Then the method of variation of parameters tells us

$$y = y_h \int \frac{q}{y_h} dt = e^{-t} \int e^t \sin(t) dt = -\frac{1}{2}\cos(t) + \frac{1}{2}\sin(t) + Ce^{-t}.$$

Problem 2. Find two different solutions to the differential equation

$$y' = y^{1/3},$$

both satisfying the initial condition $y(0) = 0$. (Note: this is an example where a solution to a differential equation exists, but is not unique.)

Solution 2. It's separable! We separate and integrate:

$$\begin{aligned} y^{-1/3} y' &= 1 \\ y^{-1/3} dy &= 1dt \\ \int y^{-1/3} dy &= \int 1dt \\ \frac{3}{2} y^{2/3} &= t + C \\ y^{2/3} &= \frac{2}{3}t + C \\ y &= \pm(\frac{2}{3}t + C)^{3/2}. \end{aligned}$$

The initial condition then tells us $C = 0$, so we see that $y = (2t/3)^{3/2}$ and $y = -(2t/3)^{3/2}$ are both solutions of this initial value problem. A third solution, even, is the constant solution $y = 0$.