Math 307 Quiz 4

October 20, 2014

Problem 1. Give an example of an initial value problem with no solution.

Solution 1. The equation

$$y' = 1/t,\ y(0) = 1$$

has no solution. In fact, the fundamental theorem of calculus tells us that any solution to $y' = 1/t$ must be of the form $y = \ln|t| + C$, for some constant C. No choice of C will allow our initial condition to be satisfied.

Problem 2. Give an example of an initial value problem with more than one solution.

Solution 2. The equation

$$y' = y^{1/3},\ y(0) = 0$$

was shown in a previous quiz to have several solutions.

Problem 3. A zombie outbreak occurs in the city of Seattle! The population P of Seattle initially is $652,404$ humans and 1 zombie (patient zero), for a total initial population $P(0) = 652,405$. Each day, the horrifying reality causes human Seattleites to flee the city to safe, remote regions of the Yukon, at a rate equal to $0.01P$, where P is the population of the city (both human and zombie combined). Furthermore, 1 percent of the remaining population H of human Seattleites is zombified each day.

(a) Set up an initial value problem describing the population P of Seattle (including both humans and zombies) as a function of time

(b) By solving (a), determine the population P of Seattle (including both humans and zombies) as a function of time

(c) Using (b) set up (but do NOT solve) an initial value problem describing the population H of human Seattleites as a function of time

Solution 3.

(a) Reading our story problem, we see that

$$\frac{dP}{dt} = \overbrace{0}^{\text{rate in}} - \overbrace{0.01P}^{\text{rate out}},$$

with the initial condition that $P(0) = 652,404$.

(b) The equation in (a) is separable. The solution is $P = Ce^{-0.01t}$, and the initial condition tells us $C = 652,404$, so that our solution is $P = 652,404e^{-0.01t}$.

(c) This is a bit trickier. We know that

$$\frac{dH}{dt} = \overbrace{0}^{\text{rate in}} - \overbrace{0.01P - rH}^{\text{rate out}}$$

where r is some constant satisfying (absent external factors, such as the fleeing) the population will decrease by 0.01 percent after 1 day. Barring external factors, we have: $\frac{dH}{dt} = -rH$, with solution $H(t) = H_0e^{-rt}$. After 1 day, this says $0.99H_0 = H_0e^{-r}$, and therefore $r = -\log(0.99)$. Thus our initial value problem is

$$\frac{dH}{dt} = -0.01(652,404)e^{-0.01t} + \log(0.99)H, \quad H(0) = 652,403.$$