Math 307 Quiz 6

November 5, 2014

Problem 1. Find the general solution to each of the following homogeneous ordinary differential equations with constant coefficients.

(a) $y'' - 3y' - 4y = 0$

(b) $2y'' + 4y' + 2y = 0$

(c) $y'' - 2y' + 2y = 0$

Solution 1.

(a) The characteristic polynomial has roots $r = 4, -1$. Therefore the general solution is
$$y = Ae^{4t} + Be^{-t}.$$

(b) The characteristic polynomial has roots $r = -1, -1$. Therefore the general solution is
$$y = Ae^{4t} + Bte^{4t}.$$

(c) The characteristic polynomial has roots $r = 1 \pm i$. Therefore the general solution is
$$y = Ae^t \cos(t) + Be^t \sin(t).$$

Problem 2. Express the complex number
$$\frac{2 + 3i}{1 - 2i}$$
in $a + ib$ form for some real numbers a, b.

Solution 2. We multiply and divide by the complex conjugate of the denominator

$$\frac{2+3i}{1-2i} = \frac{2+3i}{1-2i}\frac{1+2i}{1+2i} = \frac{(2+3i)(1+2i)}{(1-2i)(1+2i)} = \frac{-4+7i}{5} = -\frac{4}{5} + i\frac{7}{5}.$$