MATH 307: Problem Set #5

Due on: November 10, 2014

Problem 1 *Method of Undetermined Coefficients: General Solutions*

In each of the following, find the general solution of the given differential equation

(a) $y'' - 2y' - 3y = 3e^{2t}$

(b) $y'' - 2y' - 3y = -3te^{-t}$

(c) $y'' - 2y' - 3y = te^{-t} + 7e^{2t}$

(d) $y'' - 2y' - 3y = 2te^{-t} - 3e^{2t}$

(e) $y'' - 2y' - 3y = 4te^{-t} + e^{2t}$

(f) $y'' + 2y' + 5y = \sin(2t)$

(g) $y'' + 2y' + 5y = \cos(2t)$

(h) $y'' + 2y' + 5y = 4\sin(2t) + 7\cos(2t)$

(i) $y'' + 2y' = 3 + 4\sin(2t)$

(j) $y'' + 2y' + y = 2e^{-t}$

(k) $y'' + y = 3\sin(2t)$

(l) $y'' + y = t\cos(2t)$

(m) $y'' + y = 3\sin(2t) + t\cos(2t)$

(n) $y'' - y' - 2y = e^{t}$

(o) $y'' - y' - 2y = e^{-t}$

(p) $y'' - y' - 2y = \cosh(t)$ [Hint: $\cosh(t) = (e^{t} + e^{-t})/2$]

. . . .

Problem 2 *Method of Undetermined Coefficients: Initial Value Problems*

In each of the following, find the solution of the given initial value problem

(a) $y'' + 4y = t^2 + 3e^t$, $y(0) = 0$, $y'(0) = 2$

(b) $y'' - 2y' - 3y = 3te^{2t}$, $y(0) = 1$, $y'(0) = 0$

(c) $y'' + 2y' + 5y = 4e^{-t}\cos(2t)$, $y(0) = 1$, $y'(0) = 0$

. . . .

Problem 3 *Another Problem...*

Determine the general solution of

$$y'' + \lambda^2 y = \sum_{m=1}^{N} a_m \sin(m\pi t),$$

where $\lambda > 0$ and $\lambda \neq m\pi$ for $m = 1, \ldots, N$.

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Problem 4 *Differential Equations as Operators*

In this problem we indicate an alternative procedure for solving the differential equation

$$y'' + by' + cy = (D^2 + bD + c)y = g(t), \tag{1}$$

where b and c are constants, and D denotes differentiation with respect to t. Let r_1 and r_2 be the zeros of the characteristic polynomial of the corresponding homogeneous equation. These roots may be real and different, real and equal, or conjugate complex numbers.

(a) Verify that Eq (1) can be written in the factored form

$$(D - r_1)(D - r_2)y = g(t),$$

where $r_1 + r_2 = -b$ and $r_1 r_2 = c$.

(b) Let $u = (D - r_2)y$. Then show that the solution of Eq (1) can be found by solving the first order equations:

$$(D - r_1)u = g(t), \qquad (D - r_2)y = u(t).$$

. . . .

Problem 5 *Using the Previous Method...*

Using the method outlined in the previous problem, find the general solution to the following differential equations

(a) $y'' - 3y' - 4y = 3e^{2t}$

(b) $y'' + 2y' + y = 2e^{-t}$

. . . .