

Math 307 Section F
Spring 2013
Exam 1
April 24, 2013
Time Limit: 50 Minutes

Name (Print): _____

Student ID: _____

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a single, handwritten, one-sided notesheet and a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right.

1. (10 points) (a) (5 points) Find the general solution of the following first-order linear ODE using variation of parameters

$$y' = y + 7e^{2t}$$

- (b) (5 points) Find the general solution of the following IVP using the method of integrating factors

$$y' - 2y = e^{2t}, \quad y(0) = 2$$

2. Multiple Choice Section!

Directions: The multiple choice section consists of five multiple choice questions. For each question in this section, you are NOT required to justify your answer. Additionally, the following grading will be applied:

- a correct answer will receive +2 pts
- a missing answer will receive 0 pts
- a incorrect answer will receive -1 pts

with negative scores on this problem rounded to zero.

(a) (2 points) What kind of equation is the first-order ODE

$$y' = \frac{y+t}{y-t}$$

A. Linear B. Separable C. Homogeneous D. Magical

(b) (2 points) Which of the following is NOT true about the equation

$$y' = 3y - 7$$

A. It is linear B. It is separable C. It is exact D. It has int. factor $\mu(t) = e^{-3t}$

(c) (2 points) Consider the initial value problem

$$y' = f(t, y), \quad y(0) = 1.$$

For which of the following choices of function f should we not expect a unique solution?

A. $f(t, y) = t + y$ B. $f(t, y) = \frac{1}{1+y} + t$ C. $f(t, y) = \sin(ty)$ D. $f(t, y) = (1 - y)^{1/5}$

(d) (2 points) Consider the autonomous equation

$$y' = y^2(1 - y)^2$$

Which of the following is TRUE?

- A. 0 is a stable equilibrium point
- B. Both 0 and 1 are semistable equilibrium points
- C. 1 is an unstable equilibrium point
- D. 0 is a stable equilibrium point

(e) (2 points) Consider the initial value problem

$$\sin(t)y + \cos(t)y' = \frac{1}{t-3}, \quad y(\pi) = 1$$

What is the largest interval on which we should expect a solution to be defined?

A. $(-\infty, 3)$ B. $(3, -\infty)$ C. $(\pi/2, 3)$ D. $(3, 3\pi/2)$

3. (10 points) (a) (5 points) First show that

$$ye^{xy} + \cos(x) + xe^{xy}y' = 0$$

is an exact equation. Then solve it.

- (b) (5 points) Solve the equation

$$e^x dx + (e^x \cot(y) + 2y \csc(y)) dy = 0$$

by finding an integrating factor.

4. (10 points) A tank with a capacity of 500 gal originally contains 200 gal of water with 100 lbs of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixture is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.

5. (10 points) An object falling from the sky at high speed experiences a drag force proportional to its velocity. Thus its velocity v satisfies the equation

$$\frac{dv}{dt} = -g + kv^2,$$

where k is a constant depending (mostly) on the geometry of the falling object, and g is the gravitational acceleration. During the Red Bull Stratos mission, Felix Baumgartner leapt from a helium balloon and fell 39,045 meters, reaching a maximum velocity of Mach 1.25, which is 377 meters per second (approx 843 miles/hr or 1358 km/h).

- (a) (2 points) Using the fact that Baumgartner's velocity will have approached an equilibrium solution to the above autonomous equation, estimate the value of k
- (b) (6 points) Using the value for k obtained in (a), use Euler's method to complete the entries in the table below approximating Baumgartner's velocity during the first minute of his launch.

time (seconds)	velocity (meters/sec)
0	0.0
7.5	-73.575
15.0	-144.347742037
22.5	-207.136564609
30.0	-258.500922784
37.5	-297.484233931
45.0	
52.5	
60.0	

- (c) (2 points) Use the table above to get a rough idea of at what time Baumgartner broke the sound barrier. Then indicate which of the following statements are true. (Note that the speed of sound is about 343 meters per second).
- A. Baumgartner broke the sound barrier between 37.5 and 45.0 seconds.
 - B. Baumgartner broke the sound barrier between 45.0 and 52.5 seconds.
 - C. Baumgartner broke the sound barrier between 52.5 and 60.0 seconds.
 - D. Baumgartner broke the sound barrier some time after 60.0 seconds.