Math 307 Lecture 2

First Order Linear Equations!

W.R. Casper

Department of Mathematics
University of Washington

April 8, 2014

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Today!

Last time:

- First-Order Linear Equations
- Exact Equations
- Solving Exact Linear Equations

This time:

- Methods of solving First-Order Linear Equations
- Method: Integrating Factors
- Method: Variation of Parameters

Next time:

- More First-Order Linear Equations

Outline

1. First Order Linear Equations
   - What are they, and why should we care?
   - Our very first example!
2. Method of Integrating Factors
   - Exact Equations
   - Integrating Factors
   - An Example
3. Method of Variation of Parameters
   - The Method
   - An Example

Motivating Example

Example

Find the general solution of the first-order linear ODE

$$y' + \cot(t)y = \sec^2(t)$$

- Is it exact?
- Nope, so how do we solve this?
- Multiply both sides by $\sin(t)$:

$$\sin(t)y' + \cos(t)y = \sec(t)\tan(t)$$

- Now it's exact, so we know how to solve it!
- Takeaway: we made the ODE exact by multiplying both sides of the equation by a "nice" function

Motivating Example

Example

Find the general solution of the first-order linear ODE

$$y' + \cot(t)y = \sec^2(t)$$

- Is it exact?
- Nope, so how do we solve this?
- Multiply both sides by $\sin(t)$:

$$\sin(t)y' + \cos(t)y = \sec(t)\tan(t)$$

- Now it's exact, so we know how to solve it!
- Takeaway: we made the ODE exact by multiplying both sides of the equation by a "nice" function

Motivating Example

Example

Find the general solution of the first-order linear ODE

$$y' + \cot(t)y = \sec^2(t)$$

- Is it exact?
- Nope, so how do we solve this?
- Multiply both sides by $\sin(t)$:

$$\sin(t)y' + \cos(t)y = \sec(t)\tan(t)$$

- Now it's exact, so we know how to solve it!
- Takeaway: we made the ODE exact by multiplying both sides of the equation by a "nice" function

Motivating Example

Example

Find the general solution of the first-order linear ODE

$$y' + \cot(t)y = \sec^2(t)$$

- Is it exact?
- Nope, so how do we solve this?
- Multiply both sides by $\sin(t)$:

$$\sin(t)y' + \cos(t)y = \sec(t)\tan(t)$$

- Now it's exact, so we know how to solve it!
- Takeaway: we made the ODE exact by multiplying both sides of the equation by a "nice" function

Motivating Example

Example

Find the general solution of the first-order linear ODE

$$y' + \cot(t)y = \sec^2(t)$$

- Is it exact?
- Nope, so how do we solve this?
- Multiply both sides by $\sin(t)$:

$$\sin(t)y' + \cos(t)y = \sec(t)\tan(t)$$

- Now it's exact, so we know how to solve it!
- Takeaway: we made the ODE exact by multiplying both sides of the equation by a "nice" function

Motivating Example

Example

Find the general solution of the first-order linear ODE

$$y' + \cot(t)y = \sec^2(t)$$

- Is it exact?
- Nope, so how do we solve this?
- Multiply both sides by $\sin(t)$:

$$\sin(t)y' + \cos(t)y = \sec(t)\tan(t)$$

- Now it's exact, so we know how to solve it!
- Takeaway: we made the ODE exact by multiplying both sides of the equation by a "nice" function

Integrating Factors

Consider a general first order ODE

$$M(x, y) + N(x, y)y' = 0$$

Definition

A function $\mu(x, y)$ is a *integrating factor* if the equation

$$\mu(x, y)M(x, y) + \mu(x, y)N(x, y)y' = 0$$

is an exact equation.

- In other words, an integrating factor is a function that turns ODE's exact when you multiply by it!
- This integrating factor is the "nice" function from the previous example

Integrating Factors

Consider a general first order ODE

$$M(x, y) + N(x, y)y' = 0$$

Definition

A function $\mu(x, y)$ is a *integrating factor* if the equation

$$\mu(x, y)M(x, y) + \mu(x, y)N(x, y)y' = 0$$

is an exact equation.

- In other words, an integrating factor is a function that turns ODE's exact when you multiply by it!
- This integrating factor is the "nice" function from the previous example

Integrating Factors

Consider a general first order ODE

$$M(x, y) + N(x, y)y' = 0$$

Definition

A function $\mu(x, y)$ is a *integrating factor* if the equation

$$\mu(x, y)M(x, y) + \mu(x, y)N(x, y)y' = 0$$

is an exact equation.

- In other words, an integrating factor is a function that turns ODE's exact when you multiply by it!
- This integrating factor is the "nice" function from the previous example

Integrating Factors

Consider a general first order ODE

$$M(x, y) + N(x, y)y' = 0$$

Definition

A function $\mu(x, y)$ is a *integrating factor* if the equation

$$\mu(x, y)M(x, y) + \mu(x, y)N(x, y)y' = 0$$

is an exact equation.

- In other words, an integrating factor is a function that turns ODE's exact when you multiply by it!
- This integrating factor is the "nice" function from the previous example

Integrating Factors: The Linear Case

Let's think about a general first-order linear ODE

$$y' = p(x)y + q(x)$$

- Is this equation exact, do you think?
- Not usually

Question

Can we find an integrating factor for this equation, making it exact?

- Yes we can! How do we do it?

Integrating Factors: The Linear Case

Let's think about a general first-order linear ODE

$$y' = p(x)y + q(x)$$

- Is this equation exact, do you think?
- Not usually

Question

Can we find an integrating factor for this equation, making it exact?

- Yes we can! How do we do it?

Integrating Factors: The Linear Case

Let's think about a general first-order linear ODE

$$y' = p(x)y + q(x)$$

- Is this equation exact, do you think?
- Not usually

Question

Can we find an integrating factor for this equation, making it exact?

- Yes we can! How do we do it?

Integrating Factors: The Linear Case

Let's think about a general first-order linear ODE

$$y' = p(x)y + q(x)$$

- Is this equation exact, do you think?
- Not usually

Question

Can we find an integrating factor for this equation, making it exact?

- Yes we can! How do we do it?

Integrating Factors: The Linear Case

Let's think about a general first-order linear ODE

$$y' = p(x)y + q(x)$$

- Is this equation exact, do you think?
- Not usually

Question

Can we find an integrating factor for this equation, making it exact?

- Yes we can! How do we do it?

Integrating Factors: The Linear Case ∼ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Integrating Factors: The Linear Case ∼ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Integrating Factors: The Linear Case ∼ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Integrating Factors: The Linear Case $\sim$ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Integrating Factors: The Linear Case ∼ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Integrating Factors: The Linear Case ∼ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Integrating Factors: The Linear Case ∼ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Integrating Factors: The Linear Case $\sim$ Continued

- Assume that an integrating factor $\mu(x, y)$ exists
- Also assume it only depends on x: $\mu = \mu(x)$
- Then this must be exact:

$$\mu(x)y' = \mu(x)p(x)y + \mu(x)q(x)$$

- Implies $\mu'(x) = -\mu(x)p(x)$; a first-order separable ODE!
- We can solve it to get an integrating factor
- VICTORY IS ASSURED!!!

Outline

1. First Order Linear Equations
   - What are they, and why should we care?
   - Our very first example!
2. Method of Integrating Factors
   - Exact Equations
   - Integrating Factors
   - An Example
3. Method of Variation of Parameters
   - The Method
   - An Example

Method of Integrating Factors: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' - 2y = te^{3t}.$$

- Check: is it exact? (Nope!)
- So we need an integrating factor $\mu(t)$
- $\mu(t)y' - 2\mu(t)y = te^{3t}\mu(t)$ must be exact
- This means that $\mu' = -2\mu$
- Solving, we get $\mu(t) = e^{-2t}$

Method of Integrating Factors: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' - 2y = te^{3t}.$$

- Check: is it exact? (Nope!)
- So we need an integrating factor $\mu(t)$
- $\mu(t)y' - 2\mu(t)y = te^{3t}\mu(t)$ must be exact
- This means that $\mu' = -2\mu$
- Solving, we get $\mu(t) = e^{-2t}$

Method of Integrating Factors: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' - 2y = te^{3t}.$$

- Check: is it exact? (Nope!)
- So we need an integrating factor $\mu(t)$
- $\mu(t)y' - 2\mu(t)y = te^{3t}\mu(t)$ must be exact
- This means that $\mu' = -2\mu$
- Solving, we get $\mu(t) = e^{-2t}$

Method of Integrating Factors: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' - 2y = te^{3t}.$$

- Check: is it exact? (Nope!)
- So we need an integrating factor $\mu(t)$
- $\mu(t)y' - 2\mu(t)y = te^{3t}\mu(t)$ must be exact
- This means that $\mu' = -2\mu$
- Solving, we get $\mu(t) = e^{-2t}$

Method of Integrating Factors: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' - 2y = te^{3t}.$$

- Check: is it exact? (Nope!)
- So we need an integrating factor $\mu(t)$
- $\mu(t)y' - 2\mu(t)y = te^{3t}\mu(t)$ must be exact
- This means that $\mu' = -2\mu$
- Solving, we get $\mu(t) = e^{-2t}$

Method of Integrating Factors: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' - 2y = te^{3t}.$$

- Check: is it exact? (Nope!)
- So we need an integrating factor $\mu(t)$
- $\mu(t)y' - 2\mu(t)y = te^{3t}\mu(t)$ must be exact
- This means that $\mu' = -2\mu$
- Solving, we get $\mu(t) = e^{-2t}$

Method of Integrating Factors: Example ∼ Continued

Now our equation is

$$e^{-2t}y' - 2e^{-2t}y = te^{t}.$$

- Notice that $(e^{-2t}y)' = e^{-2t}y' - 2e^{-2t}$ and therefore

$$(e^{-2t}y)' = te^{t}$$
$$\int (e^{-2t}y)'dt = \int te^{t}dt$$
$$e^{-2t}y = te^{t} - e^{t} + C$$
$$y = te^{3t} - e^{3t} + Ce^{2t}$$

Method of Integrating Factors: Example ∼ Continued

Now our equation is

$$e^{-2t}y' - 2e^{-2t}y = te^t.$$

- Notice that $(e^{-2t}y)' = e^{-2t}y' - 2e^{-2t}$ and therefore

$$(e^{-2t}y)' = te^t$$
$$\int (e^{-2t}y)'dt = \int te^t dt$$
$$e^{-2t}y = te^t - e^t + C$$
$$y = te^{3t} - e^{3t} + Ce^{2t}$$

Method of Integrating Factors: Example ∼ Continued

Now our equation is

$$e^{-2t}y' - 2e^{-2t}y = te^t.$$

- Notice that $(e^{-2t}y)' = e^{-2t}y' - 2e^{-2t}$ and therefore

$$\begin{aligned} (e^{-2t}y)' &= te^t \\ \int (e^{-2t}y)'dt &= \int te^t dt \\ e^{-2t}y &= te^t - e^t + C \\ y &= te^{3t} - e^{3t} + Ce^{2t} \end{aligned}$$

Outline

1. First Order Linear Equations
   - What are they, and why should we care?
   - Our very first example!
2. Method of Integrating Factors
   - Exact Equations
   - Integrating Factors
   - An Example
3. Method of Variation of Parameters
   - The Method
   - An Example

Homogeneous Equation

Consider the first order linear ODE

$$y' = p(x)y + q(x)$$

Definition

The *homogeneous equation* associated to a first-order linear ODE is

$$y' = p(x)y$$

- CAUTION! The associated homogeneous equation isn't a homogeneous equation in the sense of last time
- Notice that this equation is *separable* (so we can solve it using the methods of last time!)

Homogeneous Equation

Consider the first order linear ODE

$$y' = p(x)y + q(x)$$

Definition

The *homogeneous equation* associated to a first-order linear ODE is

$$y' = p(x)y$$

- CAUTION! The associated homogeneous equation isn't a homogeneous equation in the sense of last time
- Notice that this equation is *separable* (so we can solve it using the methods of last time!)

Homogeneous Equation

Consider the first order linear ODE

$$y' = p(x)y + q(x)$$

Definition

The *homogeneous equation* associated to a first-order linear ODE is

$$y' = p(x)y$$

- CAUTION! The associated homogeneous equation isn't a homogeneous equation in the sense of last time
- Notice that this equation is *separable* (so we can solve it using the methods of last time!)

Homogeneous Equation

Consider the first order linear ODE

$$y' = p(x)y + q(x)$$

Definition

The *homogeneous equation* associated to a first-order linear ODE is

$$y' = p(x)y$$

- CAUTION! The associated homogeneous equation isn't a homogeneous equation in the sense of last time
- Notice that this equation is *separable* (so we can solve it using the methods of last time!)

The Method

Let y_h be a solution of the homogeneous equation associated to the linear ODE

$$y' = p(x)y + q(x)$$

- Define $v(x)$ implicitly by $y = vy_h$. Then

$$\begin{aligned} y' &= p(x)y + q(x) \\ v'y_h + vy_h' &= p(x)y + q(x) \\ v'y_h + vp(x)y_h &= p(x)y + q(x) \\ v'y_h + p(x)y &= p(x)y + q(x) \\ v'y_h &= q(x) \end{aligned}$$

$$v = \int \frac{q(x)}{y_h}dx \implies y = y_h \int \frac{q(x)}{y_h}dx$$

The Method

Let y_h be a solution of the homogeneous equation associated to the linear ODE

$$y' = p(x)y + q(x)$$

- Define $v(x)$ implicitly by $y = vy_h$. Then

$$\begin{aligned} y' &= p(x)y + q(x) \\ v'y_h + vy_h' &= p(x)y + q(x) \\ v'y_h + vp(x)y_h &= p(x)y + q(x) \\ v'y_h + p(x)y &= p(x)y + q(x) \\ v'y_h &= q(x) \\ v &= \int \frac{q(x)}{y_h}dx \quad \Longrightarrow \quad y = y_h \int \frac{q(x)}{y_h}dx \end{aligned}$$

The Method

Let y_h be a solution of the homogeneous equation associated to the linear ODE

$$y' = p(x)y + q(x)$$

- Define $v(x)$ implicitly by $y = vy_h$. Then

$$\begin{aligned} y' &= p(x)y + q(x) \\ v'y_h + vy_h' &= p(x)y + q(x) \\ v'y_h + vp(x)y_h &= p(x)y + q(x) \\ v'y_h + p(x)y &= p(x)y + q(x) \\ v'y_h &= q(x) \\ v &= \int \frac{q(x)}{y_h}dx \quad \Longrightarrow \quad y = y_h \int \frac{q(x)}{y_h}dx \end{aligned}$$

Outline

1. First Order Linear Equations
   - What are they, and why should we care?
   - Our very first example!
2. Method of Integrating Factors
   - Exact Equations
   - Integrating Factors
   - An Example
3. Method of Variation of Parameters
   - The Method
   - An Example

Method of Variation of Parameters: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' = 2y + t^2 e^{2t}$$

- The corresponding homogeneous equation is $y' = 2y$
- A solution is $y_h = e^{2t}$
- If we set $y = vy_h$, then $v = \int \frac{q(x)}{y_h} dt = \int \frac{t^2 e^{2t}}{e^{2t}} dt = \int t^2 dt = \frac{1}{3}t^3 + C$
- Since $y = vy_h$, this means $y = \frac{1}{3}t^3 e^{2t} + Ce^{2t}$

Method of Variation of Parameters: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' = 2y + t^2 e^{2t}$$

- The corresponding homogeneous equation is $y' = 2y$
- A solution is $y_h = e^{2t}$
- If we set $y = vy_h$, then $v = \int \frac{q(x)}{y_h} dt = \int \frac{t^2 e^{2t}}{e^{2t}} dt = \int t^2 dt = \frac{1}{3}t^3 + C$
- Since $y = vy_h$, this means $y = \frac{1}{3}t^3 e^{2t} + Ce^{2t}$

Method of Variation of Parameters: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' = 2y + t^2 e^{2t}$$

- The corresponding homogeneous equation is $y' = 2y$
- A solution is $y_h = e^{2t}$
- If we set $y = vy_h$, then $v = \int \frac{q(x)}{y_h} dt = \int \frac{t^2 e^{2t}}{e^{2t}} dt = \int t^2 dt = \frac{1}{3} t^3 + C$
- Since $y = vy_h$, this means $y = \frac{1}{3} t^3 e^{2t} + Ce^{2t}$

Method of Variation of Parameters: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' = 2y + t^2 e^{2t}$$

- The corresponding homogeneous equation is $y' = 2y$
- A solution is $y_h = e^{2t}$
- If we set $y = vy_h$, then
  $v = \int \frac{q(x)}{y_h} dt = \int \frac{t^2 e^{2t}}{e^{2t}} dt = \int t^2 dt = \frac{1}{3} t^3 + C$
- Since $y = vy_h$, this means $y = \frac{1}{3} t^3 e^{2t} + Ce^{2t}$

Method of Variation of Parameters: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' = 2y + t^2 e^{2t}$$

- The corresponding homogeneous equation is $y' = 2y$
- A solution is $y_h = e^{2t}$
- If we set $y = vy_h$, then $v = \int \frac{q(x)}{y_h} dt = \int \frac{t^2 e^{2t}}{e^{2t}} dt = \int t^2 dt = \frac{1}{3} t^3 + C$
- Since $y = vy_h$, this means $y = \frac{1}{3} t^3 e^{2t} + Ce^{2t}$

Method of Variation of Parameters: Example

Example

Use the method of integrating factors to find the general solution of the first-order linear ODE

$$y' = 2y + t^2 e^{2t}$$

- The corresponding homogeneous equation is $y' = 2y$
- A solution is $y_h = e^{2t}$
- If we set $y = vy_h$, then
  $v = \int \frac{q(x)}{y_h} dt = \int \frac{t^2 e^{2t}}{e^{2t}} dt = \int t^2 dt = \frac{1}{3} t^3 + C$
- Since $y = vy_h$, this means $y = \frac{1}{3} t^3 e^{2t} + Ce^{2t}$

Summary!

What we did today:

- We learned about linear equations
- We learned how to solve them with integrating factors
- We learned how to solve them with variation of parameters

Plan for next time:

- More practice solving first order linear equations

Summary!

What we did today:

- We learned about linear equations
- We learned how to solve them with integrating factors
- We learned how to solve them with variation of parameters

Plan for next time:

- More practice solving first order linear equations

Summary!

What we did today:

- We learned about linear equations
- We learned how to solve them with integrating factors
- We learned how to solve them with variation of parameters

Plan for next time:

- More practice solving first order linear equations

Summary!

What we did today:

- We learned about linear equations
- We learned how to solve them with integrating factors
- We learned how to solve them with variation of parameters

Plan for next time:

- More practice solving first order linear equations

Summary!

What we did today:

- We learned about linear equations
- We learned how to solve them with integrating factors
- We learned how to solve them with variation of parameters

Plan for next time:

- More practice solving first order linear equations

Summary!

What we did today:

- We learned about linear equations
- We learned how to solve them with integrating factors
- We learned how to solve them with variation of parameters

Plan for next time:

- More practice solving first order linear equations