Math 307 Lecture 5

Modeling Equations like a Pro!

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Department of Mathematics
University of Washington

April 8, 2014

Today!

Last time:

- Review of first order linear and separable ODEs

This time:

- First homework due Friday!!!
- Modeling first-order equations

Next time:

- Differences between linear and nonlinear equations

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Outline

Our First Example Model: Mixing fluids

Figure : Rate of pollution of a pond can be modeled by a linear ordinary differential equation

- Polluted water flows into a pond
- Volume of pond (constant): $V = 10^7$ gal
- Amount of pollutant in pond: P (metric tons)
- Toxic sludge flows in at 5×10^6 gal/yr
- Toxic sludge contains $2 + \sin(2t)$ grams of pollutant per gallon
- Lake unpolluted at $t = 0$

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Question

How does the amount of pollutant change over time?

$$\frac{dP}{dt} = \text{rate in} - \text{rate out}$$

$$\text{rate in} = \overbrace{(5 \times 10^6)}^{\text{gal. toxic sludge/yr}} \cdot \underbrace{(2 + \sin(2t))}_{\text{grams pollutant/gal}} \cdot \overbrace{(10^{-6})}^{\text{grams/ton}}$$

$$\text{rate out} = \overbrace{(5 \times 10^6)}^{\text{gal. mixed pond water/yr}} \cdot \underbrace{P(t)/V}_{\text{metric tons pollutant/gal}}$$

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- Need to solve the IVP $P(0) = 0$ and

$$\frac{dP}{dt} = 10 + 5\sin(2t) - \frac{1}{2}P.$$

- Integrating factor is $\mu(t) = e^{t/2}$
- Solution given below
- Limit behavior consists of oscillation about P=20

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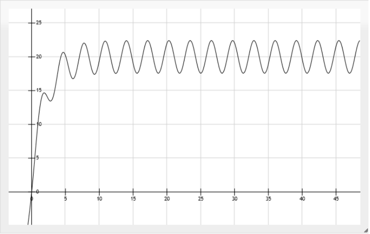

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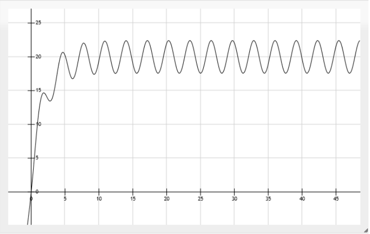

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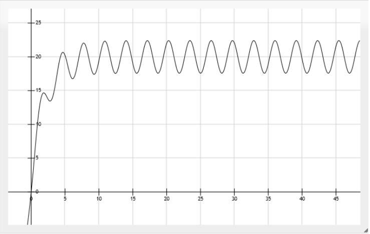

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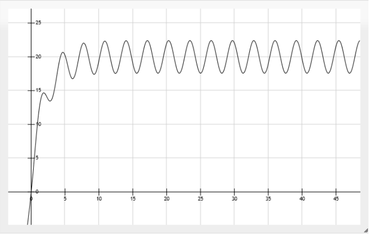

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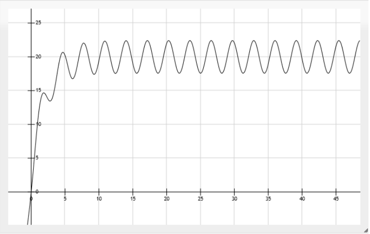

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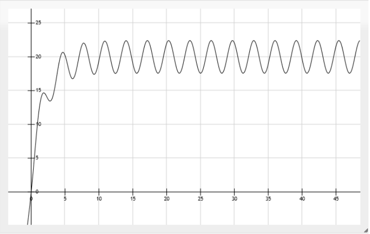

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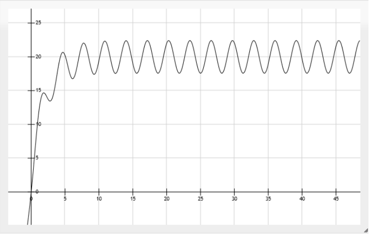

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Our Next Example Model: Radiative Heat Transfer

Figure : Radiative heat

- U abs. temp. of body
- T abs. temp. of space
- α constant depending on emissivity of body

- Stefan-Boltzmann law:

$$\frac{dU}{dt} = -\alpha(U^4 - T^4)$$

- For $U >> T$, we can approximate

$$\frac{dU}{dt} = -\alpha U^4$$

- Separable! Solution is $U = \frac{1}{(3\alpha t + C)^{1/3}}$

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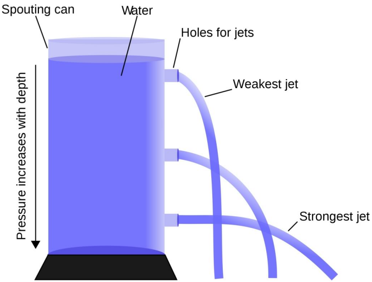
1. Linear Equation for Mixing
2. Radiative Heat Transfer
3. Holes in a Bucket
   - Torricelli's Law
   - A story problem
4. Pebble falling in Syrup
   - Stoke's Law
   - A story problem

Torricelli's Law

Figure : Water under more pressure shoots faster/farther



- Torricelli's law says
- $v = \sqrt{2gh}$
- Outflow velocity: v
- Water level above opening: h
- How fast does water leave the tank?

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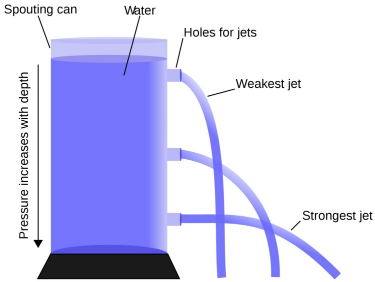

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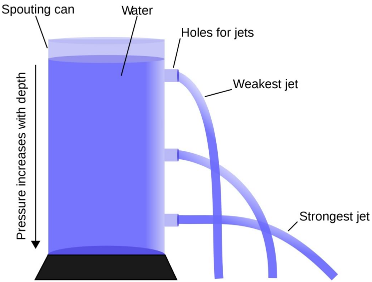

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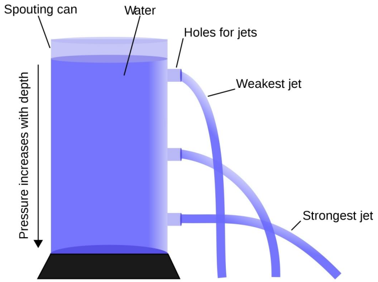

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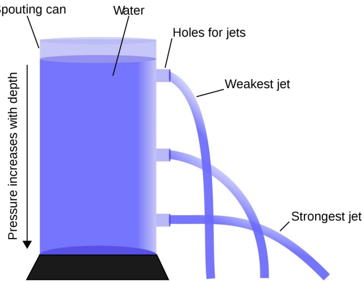

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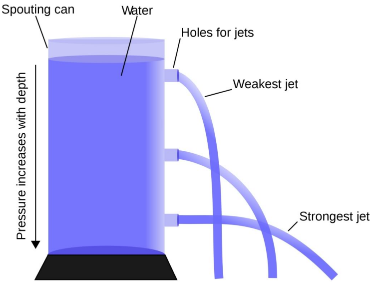

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An Old West Example

Figure : "You've got to ask yourself one question: 'Do I feel lucky?' Well, do ya punk?" – Clint Eastwood on Differential Equations

- Clint Eastwood shoots a cylindrical barrel of whiskey dead center
- bullet does not leave barrel
- barrel has height 3 feet and radius 1 foot
- he uses a .44 Magnum, the most powerful handgun in the world
- how long until the barrel is half-empty?

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- V is volume of barrel, h is height of level above bullet hole
- Barrel has cross sectional area of π
- $dV/dt = \pi \frac{dh}{dt}$
- Bullet makes a cylindrical hole with diameter 10.9 millimeters
- Cross-sectional area of $0.297 \cdot 10^{-4}\pi$
- $dV/dt = -0.297 \cdot 10^{-4}\pi\sqrt{2gh}$
- Thus

$$\pi\frac{dh}{dt} = -0.297 \cdot 10^{-4}\pi\sqrt{2gh}$$

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An Old West Example

- Separable equation!

$$\frac{1}{\sqrt{h}}\frac{dh}{dt} = -0.297 \cdot 10^{-4}\pi\sqrt{2g}$$

$$2\sqrt{h} = -(0.297 \cdot 10^{-4}\sqrt{2g})t + C$$

$$h = (C - (0.148 \cdot 10^{-4}\sqrt{2g})t)^2$$

- At $t = 0$, $h = 2$, so $C = 2$ and

$$h = (2 - (0.148 \cdot 10^{-4}\sqrt{2g})t)^2$$

- Set $h = 0$ and solve for t:

$$t = 3.0508 \times 10^4 \text{ seconds} \approx 8.475 \text{ hours}$$

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- At $t = 0$, $h = 2$, so $C = 2$ and

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- Set $h = 0$ and solve for t:

$$t = 3.0508 \times 10^4 \text{ seconds} \approx 8.475 \text{ hours}$$

An Old West Example

- Separable equation!

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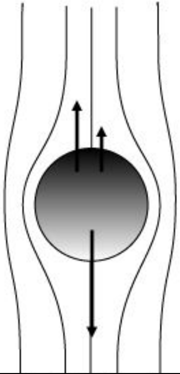
Stokes's Law

Figure : Falling pebble feels three forces



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- Ball feels three forces: buoyant force, gravitational force, viscous drag

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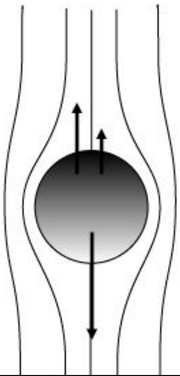

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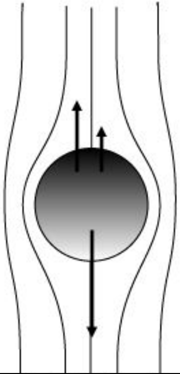

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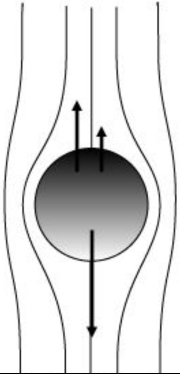

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Figure : Stokes was known for being super stoked about fluid mechanics

- buoyant force: B equals weight of fluid displaced
- viscous drag: $R = -6\pi\mu rv$ by Stokes law
- here μ quantifies how viscous the fluid is
- μ is bigger for molasses than for water
- gravitational force: $-mg$
- How fast does a pebble sink?

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3000 Leagues under the Sea

Figure : The Beatles did math in a yellow submarine...maybe

- We drop a spherical submarine into the ocean
- assume water density is approximately constant with respect to depth
- buoyant force is then $B = \frac{4}{3}\pi r^3 \rho g$
- by Newtons law: $F = m\frac{dv}{dt}$
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$$F = -mg + B + R$$

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- velocity therefore satisfies the differential equation

$$m\frac{dv}{dt} = \overbrace{-mg}^{\text{grav. force}} + \underbrace{\frac{4}{3}\pi r^3 \rho g}_{\text{buoyant force}} - \overbrace{6\pi\mu r v}^{\text{viscous drag}}$$

- Linear equation! (alt. it's separable)
- Find an integrating factor and solve. Should get:

$$v = C\exp\left(\frac{-6\pi\mu r}{m}t\right) + \frac{-mg + \frac{4}{3}\pi r^3 \rho g}{6\pi\mu r}$$

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- as we fall, we gather more speed!
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Plan for next time:

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