Math 307 Quiz 1

April 21, 2014

Problem 1. What does it mean for a differential equation of the form

$$M(x, y) + N(x, y)y' = 0$$

to be exact?

Solution 1.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Problem 2. What does it mean for $\mu(x, y)$ to be an integrating factor for the equation

$$M(x, y) + N(x, y)y' = 0$$

Solution 2. Multiplying the original differential equation by μ makes it exact.

Problem 3. Show that e^x is an integrating factor for the equation

$$y + y' = e^x$$

Then solve the equation.

Solution 3. To check that e^x is an integrating factor, we multiply the equation by it and see if it is exact. Doing so, we get the equation

$$ye^x + y'e^x = e^{2x}.$$

Putting this in M, N-form:

$$\overbrace{ye^x - e^{2x}}^{M(x,y)} + \overbrace{e^x}^{N(x,y)} y' = 0.$$

Then we calculate $M_y = e^x$ and $N_x = e^x$, so this is exact.

To solve it, we go back to the form

$$ye^x + y'e^x = e^{2x}.$$

Then we rewrite the left hand side as $(e^x y)'$, so that

$$(e^x y)' = e^{2x}.$$

Integrating both sides, we now find

$$e^x y = \frac{1}{2}e^{2x} + C.$$

The general solution is thus

$$y = \frac{1}{2}e^x + Ce^{-x}.$$

Problem 4. Find a solution to the equation $y' = y^2$ satisfying the initial condition $y(0) = 1$

Solution 4. This is separable, so we write

$$\frac{1}{y^2}y' = 1.$$

Then integrating, this becomes

$$\int \frac{1}{y^2}dy = \int 1dx,$$

which gives us

$$-\frac{1}{y} = x + C.$$

The initial condition then tells us $-1 = 0 + C$, so that $C = -1$. Thus

$$-\frac{1}{y} = x - 1.$$

Solving for y, we find

$$y = \frac{1}{1-x}.$$