Math 307 Quiz 2

April 21, 2014

Problem 1. Use the method of integrating factors to find a solution to the differential equation

$$\sin(t)y' = -\cos(t)y + 2t$$

satisfying the initial condition $y(\pi/2) = 1$.

Solution 1. Notice that the equation is already exact! Therefore, we don't need an integrating factor to solve it. We need only find the solution to the exact equation. Now we put all our y and y' terms on one side, and everything else on the other:

$$\sin(t)y' + \cos(t)y = 2t.$$

The left hand side is equal to $(\sin(t)y)'$, so we write

$$(\sin(t)y)' = 2t.$$

Integrating both sides with respect to t, we obtain

$$\sin(t)y = t^2 + C.$$

The initial condition then says $1 = \pi^2/4 + C$, so that $C = 1 - \pi^2/4$. Thus we find

$$y = t^2 \csc(t) + \left(1 - \frac{\pi^2}{4}\right) \csc(t).$$

Problem 2. Use the method of variation of parameters to find the *general* solution to the differential equation

$$y' + y = \sin(t).$$

Solution 2. We first rewrite our equation in the form $y' = p(t)y + q(t)$. Doing so, we get

$$y' = -y + \sin(t).$$

In particular, $p(t) = -1$ and $q(t) = \sin(t)$. Next we solve the corresponding homogeneous equation $y_h' = p(t)y_h$:

$$y_h' = -y_h.$$

This equation is separable, and a solution is $y_h(t) = e^{-t}$. Then the method of variation of parameters tells us that the general solution is

$$y = y_h(t) \int \frac{q(t)}{y_h(t)} = e^{-t} \int e^t \sin(t) dt = \frac{1}{2}\sin(t) - \frac{1}{2}\cos(t) + Ce^{-t}.$$

Problem 3. Find two different solutions to the differential equation

$$y' = y^{1/3},$$

both satisfying the initial condition $y(0) = 0$. (Note: this is an example where a solution to a differential equation exists, but is not unique.)

Solution 3. This equation is separable, so we separate:

$$y^{-1/3}y' = 1,$$

and then integrate

$$\int y^{-1/3} dy = \int 1 dt.$$

Therefore

$$\frac{3}{2}y^{2/3} = t + C,$$

and the initial condition $y(0) = 0$ tells us that $C = 0$, so that

$$y(t) = \left(\frac{2}{3}t\right)^{3/2}.$$

However, this is not the only solution. Another solution to this equation is the constant function $y(t) = 0$.