Math 307 Quiz 3

April 16, 2014

Problem 1. Without solving the equation, find the largest interval on which we can immediately expect a unique solution to the initial value problem

$$y' = \frac{1}{1-x}y + \csc(x), \quad y(0.1) = 42.$$

Use a sentence or two to explain your reasoning.

Problem 2. Give an example of an initial value problem with no solution.

Problem 3. Give an example of an initial value problem with a solution that is not unique.

Problem 4. Find the general solution to the differential equation

$$y + \tan(x)y' = \sec^2(x)$$

Problem 5. Initially, a tank contains 6 gal of water containing 1 lb of salt. There is water flowing into the tank through two pipes: Water containing salt is entering the tank through the first pipe at rate of 2 gal/min. Several measurements indicate that the amount of salt contained in one gallon of the incoming water is $e^{3t/2}$ lb at time t. One gallon of fresh water per minute is entering the tank through the second pipe. Finally, the well-stirred mixture is draining the tank at a rate of 3 gal/min. Determine the amount of salt at any time $t0$.