# Math 307 Lecture 11 Second-Order Homogeneous Linear ODEs with Constant Coefficients II

#### W.R. Casper

Department of Mathematics University of Washington

May 1, 2014

W.R. Casper Math 307 Lecture 11

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# Last time:

- Looked at 2nd-Order Hom. Lin. Eqns. with Constant Coefficients for the first time
- Found out how to solve them in the case that the characteristic polynomial had distinct roots
- This time:
  - Complex numbers
  - 2nd-Order Hom. Lin. Eqns. with Constant Coefficients with characteristic polynomials having complex roots

Next time:

• 2nd-Order Hom. Lin. Eqns. with Constant Coefficients with characteristic polynomials having repeated roots

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- Complex Number Basics
- Euler's Definition
- 2 Complex Roots of the Characteristic Polynomial
  - General solutions to 2nd Order Linear ODEs with Const. Coeff
  - General Case

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**Complex Number Basics** 

General Case

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Complex Number Basics Euler's Definition

# What are imaginary numbers?

- An *imaginary number* is any real number multiplied by  $\sqrt{-1}$
- Usually denote this by i
- examples: 13i,  $\sqrt{2}i$ , -4i,  $\pi i$

Figure : Imaginary numbers are very real to tigers

#### **Calvin and Hobbes**

#### by Bill Watterson

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Complex Number Basics Euler's Definition

# What are complex numbers?

#### Definition





where *a* and *b* are any real numbers.

- Any real number is a complex number also
- Any imaginary number is a complex number also

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$$2+3i$$
,  $-\sqrt{7}-\frac{1}{2}i$  and  $4+2\pi i$  are complex numbers too

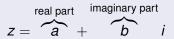
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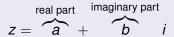
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#### Question

How do we add, subtract, multiply, and divide complex numbers by other complex numbers?

# Addition and Subtraction:

- To add complex things, just add real and imaginary parts.
- For example

$$(2+3i) + (4+2\pi i) = 6 + (3+2\pi)i$$

• Similar story for subtraction...

$$(2+3i) - (4+2\pi i) = -2 + (3-2\pi)i$$

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- To multiply complex things, we have to "foil", remembering that  $i^2 = -1$
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 $(2+3i) \cdot (4+2\pi i) = 2 \cdot 4 + 2 \cdot 2\pi i + 3i \cdot 4 + 3i \cdot 2\pi i$ = 8 + 4\pi i + 12i + 6\pi i^2 = 8 + 4\pi i + 12i - 6\pi = (8 - 6\pi) + (12 + 4\pi)i

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#### Inverses:

- We calculate the inverse of a complex number *z* = *a* + *bi* by using the complex conjugate trick
- Complex conjugate is  $z^* = a bi$

$$\frac{1}{a+bi} = \frac{1}{a+bi} \cdot 1 = \frac{1}{a+bi} \cdot \frac{(a-bi)}{(a-bi)} = \frac{a-bi}{(a+bi)(a-bi)} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i$$

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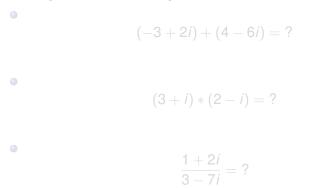
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Try it Yourself!

Complex Number Basics Euler's Definition

#### Have a go at the following calculations:



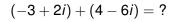
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Try it Yourself!

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Complex Number Basics Euler's Definition

#### Have a go at the following calculations:





$$\frac{1+2i}{3-7i} = ?$$

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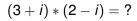
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# (-3+2i)+(4-6i)=?



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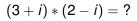
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Complex Number Basics Euler's Definition

# Outline



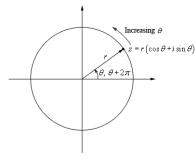
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Complex Number Basics Euler's Definition

## Complex numbers as vectors

# Figure : A complex number is a vector in the plane



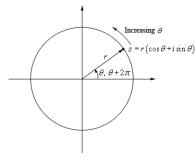
- We can "visualize" a complex number z = x + iyas a *vector*
- The tip of the vector is put at the point (*x*, *y*)
- The base of the vector is put at the origin
- Using polar coordinates, we can write

 $x = r\cos(\theta)$  $y = r\sin(\theta)$ 

Complex Number Basics Euler's Definition

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Figure : A complex number is a vector in the plane



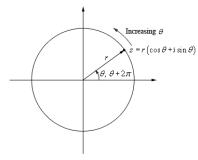
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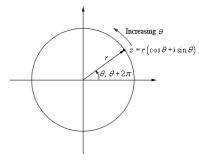
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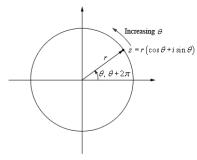
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#### Definition

If  $\theta$  is a real number, then we *define* 

$$e^{i\theta} = \cos(\theta) + i\sin(\theta).$$

#### Theorem

This definition does not break anything.

- Q: What do we mean by this?
- A:  $e^{i\theta}e^{i\phi} = e^{i(\theta+\phi)}$
- Also, this makes sense in a power-series sort of way:

$$\sum_{k=0}^{\infty} \frac{(i\theta)^k}{k!} = \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{(-1)^k \theta^{2k+1}}{(2k+1)!} i = \cos(\theta) + \sin(\theta)i = e^{i\theta}$$

#### Definition

If  $\boldsymbol{\theta}$  is a real number, then we *define* 

$$e^{i\theta} = \cos(\theta) + i\sin(\theta).$$

#### Theorem

This definition does not break anything.

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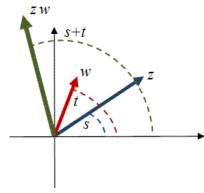
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# Complex numbers as vectors

# Figure : What does multiplication do to vectors?



- Take a complex number x + iy
- Convert (x, y) to polar:  $r = \sqrt{x^2 + y^2}$  $\theta = \tan^{-1}(y/x)$

• Then 
$$x + iy = re^{i\theta}$$
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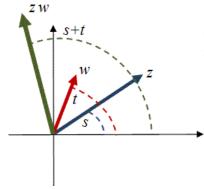
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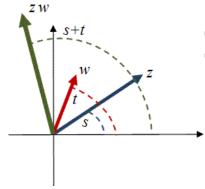
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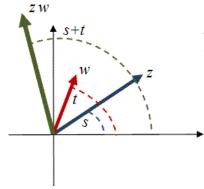
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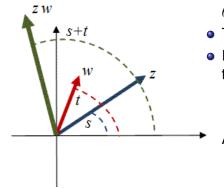
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Complex Number Basics Euler's Definition

# A few more definitions

#### Figure : SMBC



- Let  $z = re^{i\theta}$  be a complex number
- $\theta$  is called the *argument* of z
- r is called the modulus of z and is sometimes denoted as |z|
- easy exercise: show  $r^2 = z \cdot z^*$
- Q: why do we care about complex numbers?
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# Outline

#### 1 Complex Numbers

- Complex Number Basics
- Euler's Definition

#### 2 Complex Roots of the Characteristic Polynomial

- General solutions to 2nd Order Linear ODEs with Const. Coeff
- General Case

General solutions to 2nd Order Linear ODEs with Const. Coeff General Case

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# **Our First Example**

#### Example

Find the general solution of the second-order homogeneous linear ODE

$$y^{\prime\prime}-2y^{\prime}+2y=0$$

- How do we find the general solution?
- First we bluff and say we already know our solution:  $y = e^{rt}$ .
- This means  $y' = re^{rt}$  and  $y'' = r^2 e^{rt}$

General solutions to 2nd Order Linear ODEs with Const. Coeff General Case

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$$0 = y'' - 2y' + 2y$$
  
=  $r^2 e^{rt} - 2re^{rt} + 2e^{rt}$   
=  $(r^2 - 2r + 2)e^{rt}$ 

- Therefore  $r^2 2r + 2 = 0$
- Roots of the polynomial  $r^2 2r + 2$  are  $1 \pm i$
- This means  $y = e^{(1+i)t}$  and  $y = e^{(1-i)t}$  are both solutions!
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# **Our First Example**

• We only want real-valued solutions. So look at the general solution (obtained by superposition principal):

$$y = Ae^{(1+i)t} + Be^{(1-i)t}$$

• Using Euler's formula

 $y = Ae^{t}\cos(t) + iAe^{t}\sin(t) + Be^{t}\cos(t) - iBe^{t}\sin(t)$  $= (A + B)e^{t}\cos(t) + i(A - B)e^{t}\sin(t)$ 

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### **Our First Example**

- Set C = A + B and D = i(A B)
- Then *C* and *D* are arbitrary constants and the general solution is

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Complex Numbers Complex Roots of the Characteristic Polynomial General solutions to 2nd Order Linear ODEs with Const. Coeff General Case

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#### 1) Complex Numbers

- Complex Number Basics
- Euler's Definition

#### 2 Complex Roots of the Characteristic Polynomial

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#### **General Case**

$$y'' + ay' + by = 0$$

- The corresponding characteristic polynomial will be  $r^2 + ar + b$
- Roots come in conjugate pairs
- Suppose  $r^2 + ar + b$  has the root  $\alpha + \beta i$
- Then it will also have the root  $\alpha \beta i$
- Therefore the general (complex-valued) solution will look like

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#### **General Case**

$$y'' + ay' + by = 0$$

- The corresponding characteristic polynomial will be  $r^2 + ar + b$
- Roots come in conjugate pairs
- Suppose  $r^2 + ar + b$  has the root  $\alpha + \beta i$
- Then it will also have the root  $\alpha \beta i$
- Therefore the general (complex-valued) solution will look like

$$y = Ae^{(\alpha+i\beta)t} + Be^{(\alpha-i\beta)t}$$

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#### **General Case**

• Euler's formula will then give

$$y = (A+B)e^{\alpha t}\cos(\beta t) + (A-B)e^{\alpha t}\sin(\beta t)i$$

• Setting C = A + B and D = (A - B)i, the (real) general solution is

$$y = Ce^{\alpha t}\cos(\beta t) + De^{\alpha t}\sin(\beta t)$$

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Complex Numbers Complex Roots of the Characteristic Polynomial General solutions to 2nd Order Linear ODEs with Const. Coeff General Case

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# Try it Yourself!

#### Find the general solutions:

$$y^{\prime\prime}+2y^{\prime}-8y=0$$

$$y'' + 6y' + 13y = 0$$

$$y'' + 2y' + 1.25y = 0$$

General solutions to 2nd Order Linear ODEs with Const. Coeff General Case

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#### **Review!**

#### Today:

- Complex numbers and Euler's definition
- What happens when the characteristic polynomial has complex roots

Next time:

• What happens when the characteristic polynomial has repeated roots

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