Math 307 Quiz 4

May 18, 2014

Problem 1. Express the complex number

$$\frac{2+3i}{5-2i}$$

in standard form (in other words, as something of the form $a + ib$ for some real numbers a, b).

Solution 1. We use the conjugation trick to write

$$\frac{2+3i}{5-2i} = \frac{2+3i}{5-2i}\frac{5+2i}{5+2i} = \frac{(2+3i)(5+2i)}{(5-2i)(5+2i)} = \frac{4+19i}{29} = \frac{4}{29} + i\frac{19}{29}.$$

Problem 2. Find the general solution to each of the following homogeneous ordinary differential equations with constant coefficients.

(a) $y'' - 5y' + 6y = 0$

(b) $y'' - 10y' + 25y = 0$

(c) $2y'' - 3y' + 4y = 0$

Solution 2.

(a) The characteristic polynomial is z^2-5z+6, which factors as $(z-2)(z-3)$. Therefore the general solution is

$$y = C_1e^{2t} + C_2e^{3t}.$$

(b) The characteristic polynomial is $z^2 - 10z + 25$, which factors as $(z-5)(z-5)$. Therefore the general solution is

$$y = C_1e^{5t} + C_2te^{5t}.$$

(c) The characteristic polynomial is $2z^2-3z+4$. Using the quadratic formula, we calculate the roots to be $\frac{3\pm\sqrt{23}i}{4}$. Therefore the general solution is

$$y = C_1 e^{\frac{3}{4}t} \cos\left(\frac{\sqrt{23}}{4}t\right) + C_2 e^{\frac{3}{4}t} \sin\left(\frac{\sqrt{23}}{4}t\right).$$

Problem 3. Find the unique solution to the following initial value problem

(a) $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y'(1) = -1$.

Solution 3. The roots of the corresponding characteristic polynomial are $-1 \pm i$. Therefore the solution is of the form

$$y = C_1 e^{-t} \cos(t) + C_2 e^{-t} \sin(t).$$

The initial condition $y(0) = 1$ implies that

$$1 = C_1 e^{-0} \cos(0) + C_2 e^{-0} \sin(0).$$

Therefore $C_1 = 1$. The second initial condition $y'(1) = -1$ implies that

$$-1 = C_1 e^{-1} \cos(1) + C_2 e^{-1} \sin(1).$$

Then since $C_1 = 1$, we find

$$C_2 = \cot(1) - e \csc(1),$$

so that the solution to our initial value problem is given by

$$y = e^{-t} \cos(t) + [\cot(1) - e \csc(1)]\, e^{-t} \sin(t).$$