# Math 307 Quiz 6 Practice Soln

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**Problem 1.** For each of the following, determine the form of the function that should be tried, in order to find a particular solution using the method of undetermined coefficients. You do not need to solve the equation!

For example, if the equation is

$$y'' + 17y' + y = e^t,$$

then the correct response is: $y_p = Ae^t$.

Find the correct responses for each of the following equations

(a) $y'' - y = (t^3 + 2t + 1)e^t$

(b) $y'' - 3y' + 2y = t^2 e^{-2t} - 4te^{-2t} + 3e^{-2t}$

(c) $y'' + 2y' + y = t^4 e^{-t}$

(d) $y'' + 2y' + y = 3t^2 + 2t + 4$

**Solution 1.**

(a) $y_p = (At^4 + Bt^3 + Ct^2 + Dt)e^t$

(b) $y_p = (At^2 + Bt + C)e^{-2t}$

(c) $y_p = (At^6 + Bt^5 + Ct^4 + Dt^3 + Et^2)e^{-t}$

(d) $y_p = (At^2 + Bt + C)$

**Problem 2.** For each of the following, determine the complex version ("squigglyfied" version) of the differential equation. Then determine the form of the function you should try to find a particular solution to the new equation. Also describe the relationship between the particular solutions of both equations. You do not need to solve the equation! For example, if the equation is $y'' + 17y' + y = 2\cos(t)$, then the correct response is:

$$\text{squigglyfied version:} \quad \widetilde{y}'' + 17\widetilde{y}' + \widetilde{y} = 2e^{it}.$$

$$\text{form to try:} \quad \widetilde{y}_p = Ae^{it}$$

$$\text{relation to } y_p\text{:} \quad y_p = \mathrm{Re}(\widetilde{y}_p)$$

Find the correct responses for each of the following equations

(a) $y'' + 2y' + y = t^2\cos(t)$

(b) $y'' + y = 2t\sin(t)$

(c) $y'' - 2y' + 2y = te^t\sin(t)$

**Solution 2.**

(a) $\widetilde{y}'' + 2\widetilde{y}' + \widetilde{y} = t^2e^{it}$, $\widetilde{y}_p = (At^2 + Bt + C)e^{it}$, $y_p = \mathrm{Re}(\widetilde{y}_p)$.

(b) $\widetilde{y}'' + \widetilde{y} = 2te^{it}$, $\widetilde{y}_p = (At^2 + Bt)e^{it}$, $y_p = \mathrm{Im}(\widetilde{y}_p)$.

(c) $\widetilde{y}'' - 2\widetilde{y}' + 2\widetilde{y} = te^{(1+i)t}$, $\widetilde{y}_p = (At^2 + Bt)e^{(1+i)t}$, $y_p = \mathrm{Im}(\widetilde{y}_p)$.

**Problem 3.** Set up, but do not solve, an initial value problem describing the motion of the mass-spring system described in the following story problem. Attaching a weight of 3 lbs. to a certain spring stretches it 6 inches. A dampening device is also attached, and the device exerts a drag force of 1 lb when the spring is moving 2 feet per second. The spring is stretched an additional two inches and then thrown downward at an initial velocity of 0.2 feet per second.

**Solution 3.** The units we will try to put everything in will be those related to feet, pounds, and seconds. The mass of the weight is

$$m = \frac{3 \text{ lbs}}{32 \text{ ft/sec}^2} = \frac{3}{32}\text{lbs} \cdot \text{ft/sec}^2.$$

The force of drag is proportional to the velocity, and since this force is 1 lb. when the spring is moving 2 ft./sec. the coefficient of drag is

$$\gamma = \frac{1 \text{ lbs}}{2 \text{ ft/sec}} = \frac{1}{2}\text{lbs} \cdot \text{sec/ft.}$$

Also a force of 3 lbs stretches a spring 6 inches, ie. 1/2 a foot. Therefore the spring constant of the spring is

$$k = \frac{3 \text{ lbs}}{(1/2) \text{ ft}} = 6\text{lbs/ft.}$$

The initial conditions may be read from the problem then as $u(0) = (1/6)$ ft and $u'(0) = 0.2$ ft/sec. The differential equation describing the motion of the spring is

$$mu'' + \gamma u' + ku = 0.$$

Therefore the initial value problem we have is

$$\frac{3}{32}u'' + \frac{1}{2}u' + 6u = 0, \quad u(0) = \frac{1}{6}, \quad u'(0) = 0.2$$