Math 307 Quiz 6

May 19, 2014

Problem 1. For each of the following, determine the correct form of particular solution y_p to try in the method of undetermined coefficients.

- (a) $y'' + y = t^2 e^{-2t} 4t e^{-2t} + 3e^{-2t}$
- (b) $y'' 2y' + y = t^3 e^t$
- (c) $y'' y = t^4 e^{-t}$
- (d) $y'' + 2y' = 3t^2 + 2t + 4$

Solution 1.

- (a) $y_p = (At^2 + Bt + C)e^{-2t}$
- (b) $y_p = (At^5 + Bt^4 + Ct^3 + Dt^2)e^t$
- (c) $y_p = (At^5 + Bt^4 + Ct^3 + Dt^2 + Et)e^{-t}$
- (d) $y_p = At^3 + Bt^2 + Ct$

Problem 2. Suppose that \tilde{y}_p is a particular solution to the differential equation $\gamma'' + 22\gamma' + 4\gamma - e^{(17+4i)t}$

$$\widetilde{y}'' + 23\widetilde{y}' + 4\widetilde{y} = te^{(17+4i)t}.$$

Fill in the blanks in the equation

$$y_p = _$$
____Re(\tilde{y}_p) + _____Im(\tilde{y}_p)

so that y_p is a particular solution to the equation

$$y'' + 23y' + 4y = 3te^{17t}\cos(4t) + 7te^{17t}\sin(4t).$$

Solution 2.

$$y_p = 3\operatorname{Re}(\widetilde{y}_p) + 7\operatorname{Im}(\widetilde{y}_p).$$

Problem 3. Consider the differential equation

$$y'' - 4y = t\cos(2t)$$

The complex differential equation that we associate with this is

$$\widetilde{y}'' - 4\widetilde{y} = te^{2it}.$$

- (a) For the second equation, which form of \tilde{y}_p should we use in the method of undetermined coefficients?
- (b) How is the particular solution \tilde{y}_p to the complex equation related to the particular solution y_p of the original equation?

Solution 3.

(a)
$$\widetilde{y}_p = (At+B)e^{2it}$$

(b)
$$y_p = \operatorname{Re}(\widetilde{y}_p)$$