Math 307 Section M
Spring 2015
Midterm 1
April 22, 2015
Time Limit: 50 Minutes

Name (Print): ____________________

Student ID: ____________________

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a single, handwritten, one-sided notesheet and a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. Solve the following initial value problems

   (a) (5 points) Solve the following initial value problems:

$$\frac{dy}{dx} = xe^x/y^2 + x/y^2, \quad y(0) = 1$$

   (b) (5 points)

$$y' = \frac{x+y}{x-y}, \quad y(0) = 1$$

2.

(a) (5 points) Find the general solution to the linear equation

$$y' = 3y + \cos(2x)$$

using the method of integrating factors.

(b) (5 points) Find the general solution to the linear equation

$$y' = -\cot(x)y + \cos(x) + 3$$

using the method of variation of parameters.

3. (a) (2 points) Give an example of an initial value problem with no solution

   (b) (2 points) Give an example of an initial value problem with more than one solution

   (c) (2 points) Without solving the equation, on what interval can we expect a unique solution to the initial value problem

$$\sin(x)y' = \cos(x)y + \frac{1}{1-x}, \quad y(\pi/2) = 1.$$

   (d) (2 points) Write the complex number $e^{i\pi/6}$ in $a + ib$ form

   (e) (2 points) Write the complex number $\frac{2+3i}{3-2i}$ in $a + ib$ form

4. (a) (5 points) Find an integrating factor for the equation

$$ye^{xy} + xy(y+1)e^{xy} + (x + x^2y)e^{xy}y'.$$

You do NOT need to solve it.

(b) (5 points) Show that the equation

$$2x + 3y - \frac{2x}{(x^2+y^2)^2} + \left(3x - \frac{2y}{(x^2+y^2)^2} - y\right)y' = 0$$

is exact. Then find a family of solutions.

5. (10 points) **Rubber Band Ball Drop**

In March 2003, the worlds largest rubber band ball had a mass of roughly 1000 kg and a diameter of 1.4 meters. For science, it was dropped from an airplane at a height of about 1.6 kilometers from the surface. During its descent, the ball experienced two forces: the gravitational force F_g given by

$$F_g = -mg$$

and the force of drag

$$F_d = \frac{1}{2}\rho v^2 C_D \pi r^2.$$

Here, the ρ is the density of air ($\rho = 1.225$ kg/m^3), C_D is the drag coefficient ($C_D = 0.4$), r is the radius, m is the mass and g is the acceleration due to gravity ($g = 9.81$ m/s^2).

(a) Set up an initial value problem describing the vertical velocity of the rubber band ball as a function of time

(b) Find a solution to the differential equation in (a) and use it to determine the *terminal velocity* v_t of the rubber band ball – ie. the velocity of the ball at large times (it appears as a horizontal asymptote of $v(t)$)

(c) In this situation, the velocity of the ball very quickly approaches the terminal velocity, so a good estimation for the velocity of the ball during most of the descent is constant v_t. By approximating that the ball's velocity is always v_t, estimate the time it takes the ball to hit the ground.