

Math 307 Section M
Spring 2015
Exam 2
May 22, 2015
Time Limit: 50 Minutes

Name (Print): _____

Student ID: _____

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a single, handwritten, one-sided notesheet and a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
Total:	70	

Do not write in the table to the right.

1. (10 points) Solve the following initial value problem

$$y'' + 6y' + 9y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

2. Propose a Solution Section!

Directions: The “Propose a Solution” section consists of five linear nonhomogeneous equations. For each of these equations, write down the type of function y (with undetermined coefficients) you would try, in order to get a particular solution. *You do NOT need to solve the equations* For example, if the equation were

$$y'' + 2y' + y = e^t,$$

a *correct answer* would be

$$y = Ae^t,$$

and *incorrect answers* would include

$$y = (At + B)e^t, \quad y = At^2e^{2t}, \quad y = Ae^{3t}, \quad y = A\pi^t$$

Each part is worth 2pts:

(a) (2 points)

$$4y'' + 5y' + y = t^3e^{2t}$$

(b) (2 points)

$$y'' + y' + 3y = (t + 1)e^{-t}$$

(c) (2 points)

$$y'' + 4y' - 5y = e^t$$

(d) (2 points)

$$y'' + 12y' = t + 17$$

(e) (2 points)

$$y'' - 4y' + 4y = e^{2t}$$

3. (10 points) Find a family of solutions to the differential equation

$$y' = -\frac{3xy + y^2}{x^2 + xy}.$$

4. (10 points)

(a) (4 points) Find a particular solution to the equation

$$y'' + 2y' + y = \cos(5t)$$

(b) (2 points) Find a particular solution to the equation

$$y'' + 2y' + y = \sin(5t)$$

(c) (2 points) Find a particular solution to the equation

$$y'' + 2y' + y = 3 \cos(5t) - \sin(5t)$$

(d) (2 points) Write down the general solution to the equation

$$y'' + 2y' + y = 3 \cos(5t) - \sin(5t)$$

5. (10 points) Given that $y_1 = t^2$ is a solution to the differential equation

$$t^2 y'' - 3ty' + 4y = 0,$$

use the method of reduction of order to find the general solution of the equation.

6. (20 points) A mass of 3 kg is connected to a spring, stretching it 2.4525 meters to a new equilibrium length. The spring is then stretched downward $\sqrt{2}/2$ meters and then thrown downward at a velocity of 1 meter per second. (Note: gravitational acceleration $g = 9.81 \text{ m/s}^2$)
- (a) (5 points) Set up an initial value problem describing the position of the spring relative to its equilibrium position $u(t)$.
- (b) (5 points) Solve the IVP of (a) to find $u(t)$.
- (c) (5 points) Determine the amplitude, period, and phase of the resultant motion.
- (d) (5 points) Sketch a graph of $u(t)$, and indicate how the amplitude, period, and phase affect the shape of the graph.