

# Weekly Homework 1

Due: Monday April 13, 2015

April 10, 2015

**Problem 1 (Solutions To Differential Equations).** For each of the following, show whether or not the specified function is a solution to the corresponding differential equation.

(a)  $y'''' + y''' + y' - y = 0$ ,  $y(x) = \cos(x)$

(b)  $\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + 6u \frac{\partial u}{\partial x} = 0$ ,  $u(x, t) = \frac{1}{2} \operatorname{csech}^2 \left[ \frac{\sqrt{c}}{2} (x - ct - a) \right]$

(c)  $y'' - y = 0$ ,  $y(x) = \sinh(x)$

**Problem 2 (Solving differential equations).** For each of the following differential equations, do the following

(i) Identify the type of differential equation

(ii) Find the “general solution”

(a)  $y' = 2y + 3$

(b)  $y' = \frac{x^2 - y^2}{x + y}$

(c)  $\sin(u) \frac{du}{dt} = \cos(u)/(1 + t^2)$

(d)  $\frac{dy}{dt} = \frac{t^2 - y^2}{ty}$

(e)  $(3x - 4y)dy = (2x + 7y)dx$

(f)  $\frac{dy}{dt} + y/t = 6 \cos(4t)$

(g)  $y' + y = \cos(t)$

(h)  $y' = 1 - y^3$

**Problem 3 (Waaaaait a minute!).** Explain what is wrong with the following argument:

Consider the differential equation

$$y' = 1 - 2y$$

Integrating both sides, we get the equation

$$y = y - y^2 + C.$$

Simplifying this, we get the solution  $y^2 = C$  meaning that

$$y = \pm\sqrt{C}.$$

**Problem 4 (Slope fields).** For each of the following initial value problems

- (i) Plot the slope field
  - (ii) Based on the plot of the slope field, predict the behavior of a solution to the IVP at large values of  $t$
  - (iii) Explicitly solve the IVP
  - (iv) Based on the explicit solution of the IVP, determine the behavior at large values of  $t$
- (a)  $y' = y(1 - y^2)$ ,  $y(0) = 1$
  - (b)  $y' = y(1 - y^2)$ ,  $y(0) = 1/2$
  - (c)  $y' = y(1 - y^2)$ ,  $y(0) = 3/2$

**Problem 5 (Second order equations).** Consider the second order differential equation

$$y'' - y = 0$$

- (a) Show that the change of variables  $z = y' + y$  in the above second-order equation transforms it into the first order equation

$$z' - z = 0$$

- (b) Find the general solution of the first-order equation of (a)
- (c) By substituting the value of  $z$  back into the equation  $z = y' + y$ , find the value of  $y$ . Your final answer for  $y$  should involve *two* arbitrary constants.

**Problem 6 (Solving Initial Value Problems).** Find a solution to each of the following initial value problems

- (a)  $y' = x \cos(y)$ ,  $y(0) = 1$
- (b)  $y' = e^x + y$ ,  $y(1) = 2$

(c)  $\frac{dy}{dt} + 2y = te^{-2t}$ ,  $y(1) = 0$

(d)  $xy' + 2y = \sin(x)$ ,  $y(\pi/2) = 1$

**Problem 7 (An almost homogeneous equation).** Consider the differential equation

$$y' = x \cos(y/x) + y/x$$

(a) Explain why this is not a homogeneous differential equation

(b) Find the general solution of the differential equation.