MATH 307: Problem Set #4

Due on: May 11, 2015

Problem 1 *Wronskian*

For each of the following collections of functions, use the Wronskian to show that the collection is linearly independent.

(a) e^{2x}, e^{3x}

(b) e^x, xe^x

(c) $1, x, x^2$

. . . .

Problem 2 *Homogeneous ODEs with Const. Coeffs: Distinct Roots*

In each of the following, find the general solution of the given differential equation

(a) $y'' + 3y' + 2y = 0$

(b) $2y'' - 3y' + y = 0$

(c) $y'' - 2y' - 2y = 0$

. . . .

Problem 3 *Homogeneous IVPs with Const. Coeffs: Distinct Roots*

In each of the following, find the solution of the IVP

(a) $y'' + 4y' + 3y = 0, \quad y(0) = 2,\ y'(0) = -1$

(b) $y'' + 3y' = 0, \quad y(0) = -2,\ y'(0) = 3$

. . . .

Problem 4 *Complex Number Problems*

In each of the following, rewrite the expression in the form $a + ib$

(a) e^{2-3i}

(b) $e^{2-(\pi/2)i}$

(c) π^{-1+2i}

. . . .

Problem 5 *Homogeneous ODEs with Const. Coeffs: Complex Roots*

In each of the following, find the general solution of the ODE

(a) $y'' - 2y' + 6y = 0$

(b) $y'' + 2y' + 2y = 0$

(c) $y'' + 4y' + 6.25y = 0$

. . . .

Problem 6 *Homogeneous IVPs with Const. Coeffs: Complex Roots*

In each of the following, find the solution of the IVP

(a) $y'' + 4y = 0, \quad y(0) = 0, \, y'(0) = 1$

(b) $y'' + 4y' + 5y = 0, \quad y(0) = 1, \, y'(0) = 0$

. . . .

Problem 7 *Homogeneous ODEs with Const. Coeffs: Repeated Roots*

In each of the following, find the general solution of the ODE

(a) $9y'' + 6y' + y = 0$

(b) $4y'' + 12y' + 9y = 0$

(c) $y'' - 6y' + 9y = 0$

(d) $25y'' - 20y' + 4y = 0$

. . . .

Problem 8 *Reduction of Order*

In each of the following, use the method of reduction of order to find a second solution of the ode

(a) $t^2y'' + 2ty' - 2y = 0, \;\; t > 0$ (one solution is $y(t) = t$)

(b) $(x-1)y'' - xy' + y = 0, \;\; x > 1$ (one solution is $y(x) = e^x$)

. . . .

Problem 9 *Euler-Cauchy Equation*

A second-order Euler-Cauchy equation is a second-order homogeneous linear ordinary differential equation with non-constant coefficients of the form

$$at^2\frac{d^2y}{dt^2} + bt\frac{dy}{dt} + cy = 0, \tag{1}$$

where a, b, c are constants with $a \neq 0$. Due to it's regular form, the Euler-Cauchy equation may be transformed into a homogeneous linear ordinary differential equation with constant coefficients, by means of an appropriate variable substitution.

Consider the variable substitution $t = e^u$

(a) Show that

$$\frac{dy}{du} = t\frac{dy}{dt}$$

(b) Show that

$$\frac{d^2y}{du^2} = t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt}$$

(c) Using (a) and (b), show that the Euler-Cauchy Equation (1) is equivalent to the second-order linear ordinary differential equation with constant coefficients

$$a\frac{d^2y}{du^2} + (b-a)\frac{dy}{du} + cy = 0.$$

. . . .

Problem 10 *Euler-Cauchy Equation Practice*

Find the general solution to each of the following equations

(a) $t^2y'' + 4ty' + 2y = 0, \;\; t > 0$

(b) $3t^2y'' + 7ty' - 4y = 0, \;\; t > 0$

. . . .

Problem 11 *Higher-Order ODE's*

In this class, we will mostly stick with first and second-order equations. However, it is important to recognizer that many of the methods we outline for first and second order equations naturally generalize to the case of higher-order equations. For each of the following equations, do your best to extend a method we have learned previously, in order to find the general solution.

(a) $y'''' + y = 0$

(b) $y''' - 3y'' - 3y' + y = 0$

(c) $y''' - y'' - y' + y = 0$

(d) $t^3 y''' + 3t^2 y'' + ty' + y = 0$

. . . .