MATH 307: Problem Set #6

Due on: May 27, 2015

Problem 1 *Trigonometric Forcing*

Find a particular solution to each of the following differential equations

(a) $y'' + 2y' + y = \sin(t)$

(b) $y'' + 2y' + y = \cos(t)$

(c) $y'' + 2y' + y = 3\sin(t) + 2\cos(t)$

(d) $y'' + y = \cos(t)$

(e) $y'' + y = e^{-2t}\sin(t)$

(f) $y'' + 2y' + 2y = e^{-t}\cos(t)$

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Problem 2 *Trigonometry Exercise*

In each of the following, determine ω_0, R, δ so as to write the given expression in the form $u = R\cos(\omega_0 t - \Delta)$.

(a) $u = 3\cos(2t) + 4\sin(2t)$

(b) $u = 4\cos(3t) - 2\sin(3t)$

(c) $u = -2\cos(\pi t) - 3\sin(\pi t)$

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Problem 3 *A Spring Problem*

A mass weighing 3 lbs stretches a spring 3 inches. If the mass is pushed upward, contracting the spring a distance of 1 in., and then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period, amplitude, and phase of motion.

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Problem 4 *Another Spring Problem*

A spring is stretched 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is 5 m/s. If the mass is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm/s, determine its position u at any time t. Find the quasifrequency μ and the ratio of μ to the natural frequency corresponding to undamped motion.

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Problem 5 *LCR Circuit Problem*

If a series circuit has a capacitor of $C = 0.8 \times 10^{-6}$ F and an inductor of $L = 0.2$ H, find the smallest value of the resistance R so that the circuit is critically damped.

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Problem 6 *A Forced Spring Problem*

A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10\sin(t/2)$ N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s, formulate the initial value problem describing the motion of the mass.

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Problem 7 *Another Forced Spring Problem*

If an undamped spring-mass system with a mass that weighs 6 lb and a spring constant of 41 lb/in is suddenly set in motion at $t = 0$ by an external force of $4\cos(7t)$ lb, determine the position of the mass at any time and draw a graph of the displacement versus t.

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Problem 8 *A Third Forced Spring Problem*

A mass that weighs 8 lb stretches a spring 6 inches. The system is acted on by an external force of $8\sin(8t)$ lb. If the mass is pulled down 3 in and then released, determine the position of the mass at any time. Determine the first four times at which the velocity of the mass is zero.

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Problem 9 *Continuity Problem*

In each of the following sketch a graph of the function and determine whether it is continuous, piecewise continuous, or neither on the interval $0 \leq t \leq 3$.

(a)

$$f(t) = \begin{cases} t^2, & 0 \leq t \leq 1 \\ 1, & 1 < t \leq 2 \\ 3 - t, & 2 < t \leq 3 \end{cases}$$

(b)

$$f(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 3 - t, & 1 < t \leq 2 \\ 1, & 2 < t \leq 3 \end{cases}$$

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