MATH 307: Problem Set #7

Due on: June 5, 2015

Problem 1   *Laplace Transforms*

In each of the following, deptermine the Laplace transform of the given function $f(t)$.
Note that n is a positive integer and a is a real constant.

(a) $f(t) = \cosh(at)$ [Recall that $\cosh(at) = (e^{at} + e^{-at})/2$]

(b) $f(t) = te^{at}$

(c) $f(t) = t\sin(at)$

(d) $f(t) = t^n e^{at}$

$$\cdots\cdots\cdots$$

Problem 2   *Inverse Laplace Transforms*

In each of the following, find the inverse Laplace transform of the given function

(a) $F(s) = \frac{3}{s^2+4}$

(b) $F(s) = \frac{4}{(s-1)^3}$

(c) $F(s) = \frac{2}{s^2+3s-4}$

(d) $F(s) = \frac{3s}{s^2-s-6}$

(e) $F(s) = \frac{2s+1}{s^2-2s+2}$

(f) $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$

(g) $F(s) = \frac{1-2s}{s^2+4s+5}$

(h) $F(s) = \frac{2s-3}{s^2+2s+10}$

$$\cdots\cdots\cdots$$

Problem 3   *Solving IVPs*

In each of the following, use Laplace transforms to solve the initial value problem

(a) $y'' + 3y' + 2y = 0$, $y(0) = 1$, $y'(0) = 0$

(b) $y'' - 2y' + 2y = 0$, $y(0) = 0$, $y'(0) = 1$

(c) $y'' + 2y' + 5y = 0$, $y(0) = 2$, $y'(0) = -1$

(d) $y'' + \omega^2 y = \cos(2t)$, $y(0) = 1$, $y'(0) = 0$ (Here, ω is a constant and $\omega^2 \neq 4$.)

$\cdots\cdots\cdots$

Problem 4   *Step Functions*

In each of the following, sketch a graph of the given function on the interval $t \geq 0$

(a) $g(t) = u_1(t) + 2u_3(t) - 6u_4(t)$

(b) $g(t) = (t-1)u_1(t) - 2(t-2)u_2(t) + (t-3)u_3(t)$

$\cdots\cdots\cdots$

Problem 5   *Brackets to Step Functions*

In each of the following, sketch a graph of the given function on the interval $t \geq 0$, and then convert the function from bracket form to step function form

(a)
$$f(t) = \begin{cases} 1 & \text{if } 0 \leq t < 2 \\ e^{-(t-2)} & \text{if } t \geq 2 \end{cases}$$

(b)
$$f(t) = \begin{cases} t & \text{if } 0 \leq t < 1 \\ t-1 & \text{if } 1 \leq t < 2 \\ t-2 & \text{if } 2 \leq t < 3 \\ 0 & \text{if } t \geq 3 \end{cases}$$

$\cdots\cdots\cdots$

Problem 6 *Laplace Transforms of Step Functions*

Find the Laplace transform of the given function

(a)

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 2 \\ (t-2)^2 & \text{if } t \geq 2 \end{cases}$$

(b)

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 1 \\ t^2 - 2t + 2 & \text{if } t \geq 1 \end{cases}$$

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Problem 7 *More Inverse Laplace Transforms*

Find the Inverse Laplace transform of the given function

(a) $F(s) = \frac{e^{-2s}}{s^2+s-2}$

(b) $F(s) = \frac{2e^{-2s}}{s^2-4}$

(c) $F(s) = \frac{e^{-s}+e^{-2s}-e^{-3s}-e^{-4s}}{s}$

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Problem 8 *Solutions to IVPs with Discontinuous Forcing*

In each of the following problems, find the solution of the given initial value problem

(a) $y'' + 2y' + 2y = h(t)$, $y(0) = 0$, $y'(0) = 1$, $h(t) = \begin{cases} 0 & \text{if } 0 \leq t < \pi \\ 1 & \text{if } \pi \leq t < 2\pi \\ 0 & \text{if } 2\pi \leq t \end{cases}$

(b) $y'' + 3y' + 2y = f(t)$, $y(0) = 0$, $y'(0) = 0$, $f(t) = \begin{cases} 1 & \text{if } 0 \leq t < 10 \\ 0 & \text{if } t \geq 10 \end{cases}$

(c) $y'' + y = u_{3\pi}(t)$, $y(0) = 1$, $y'(0) = 0$.

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