Math 307 Section F **Name (Print):** ____________________
Fall 2014
Exam 1 **Student ID:** ____________________
October 20, 2014
Time Limit: 50 Minutes

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a single, handwritten, one-sided notesheet and a *basic* calculator.

Note also that the LAST PAGE has a couple useful integrals!!

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right.

1. (10 points) (a) (5 points) Find a particular solution of the IVP

$$xy' = -\cos(y), \quad y(1/2) = 0.$$

(b) (5 points) Find the general solution of the differential equation

$$xy' + 2y = \sin(x).$$

2. **Multiple Choice Section!**

Directions: The multiple choice section consists of five multiple choice questions. You are NOT required to justify your answer.

(a) (2 points) Which of the following initial value problems could have more than one solution?

A.
$$y' + y^2 = -3xy + \cos(x), \quad y(\pi) = 2\pi^2.$$

B.
$$xy' = 2xy - 3, \quad y(1) = 1.$$

C.
$$y' = y^{1/3}, \quad y(0) = 1.$$

D.
$$y' = |1 - y| + 1, \quad y(0) = 1.$$

(b) (2 points) The equation
$$y' = 3y\frac{xy^2 + yx^2}{x + y}$$
is which type of equation?

A. Linear B. Homogeneous C. Seperable D. Exact

(c) (2 points) What is the largest interval we should expect a unique solution to the initial value problem
$$(3 - x)y' = \sin(x)y + \tan(x), \quad y(\pi) = 0$$

A. $(\pi/2, 3)$ B. $(3, 3\pi/2)$ C. $(\pi/2, 3\pi/2)$ D. $(-\pi/2, \pi/2)$

(d) (2 points) Which of the following statements is TRUE?

A. The general solution of $y' = \frac{1}{y(1-y)(1+y)}$ is $y = \pm\sqrt{\frac{Ce^x}{Ce^x - 1}}$

B. An initial value problem must have at least one solution.

C. If $\mu(x, y)$ is an integrating factor for the equation $M(x, y) + N(x, y)y' = 0$, then $42\mu(x, y)$ is also an integrating factor for the same equation.

D. A family of solutions to $y' = 2x + 2y$ is given by $y = x^2 + y^2 + C$.

(e) (2 points) Which of the following statements is TRUE about the equation
$$y' = y(e^y - e^{-y})(1 - y^2)$$

A. $y = 0$ is a semistable equilibrium

B. $y = 1$ is an unstable equilibrium

C. $y = -1$ is a semistable equilibrium

D. $y = 0$ is a stable equilibrium solution

3. (10 points) (a) (5 points) Show that the following equation is exact. Then find a family of solutions.

$$e^x \sin(y) - 2y\sin(x) + (e^x \cos(y) + 2\cos(x))y' = 0.$$

(b) (5 points) Find an integrating factor for the following differential equation (you do NOT need to solve the equation).

$$3x^2y + 2xy + y^3 + (x^2 + y^2)y' = 0.$$

4. (10 points) Newton's law of cooling states that the temperature $u(t)$ of an object changes at a rate proportional to $u(t) - T(t)$, where $T(t)$ is the surrounding temperature at time t. To put this another way,

$$u'(t) = -k(u(t) - T(t)),$$

for some constant k which in general depends on the material properties of the object and its surroundings. At noon, a differential equations student fills a bird bath outdoors with water at u_0 degrees F. The temperature outside is given by $T(t) = 65 + 5\cos(\pi t/12)$, where t is given in hours since noon.

(a) Find an equation for $u(t)$ (your answer should involve k and u_0).

(b) After two hours have passed, the temperature of the water in the bath "acclimates", and any terms of the form e^{-kt} in your solution from (a) are very, very small (and may be taken to be zero). With this in mind, write an approximate expression for $u(t)$, for $t > 2$ hours. Does the temperature after two hours depend on the starting temperature u_0?

(c) Suppose that the next day, the differential equations student measures the temperature of the water in the bird bath to be 0.1 degree less than the surrounding temperature. Using your approximation from (b), determine the value of k.

5. (10 points)

(a) (5 points) Find a family of solutions of the following differential equation

$$y' = \frac{x - y}{x + y}.$$

(b) (5 points) The population of creatures on a radioactive island consists of humans and zombies. Let h denote the fraction of the population which is humans and z denote the fraction of the population which is zombies, so that $h + z = 1$. Humans that come in contact with zombies end up as zombies themselves, so that the rate of change of the zombie population is equal to the number of such contacts. Furthermore, assume both humans and zombies move about the island freely, so that the number of contacts is proportional to the product of h and z, with proportionality constant k. Assuming that 0.99 of the population is human initially, set up (but do not solve) an initial value problem describing the population of zombies as a function of time.

SOME HELPFUL INTEGRALS:

$$\int e^{ax}\cos(bx)dx = \frac{a}{a^2+b^2}e^{ax}\cos(bx) + \frac{b}{a^2+b^2}e^{ax}\sin(bx) + C.$$

$$\int e^{ax}\sin(bx)dx = \frac{-b}{a^2+b^2}e^{ax}\cos(bx) + \frac{a}{a^2+b^2}e^{ax}\sin(bx) + C.$$