Weekly Homework 1

Due: Friday, Jan 15 2016

January 8, 2016

Problem 1 (Solutions To Differential Equations). For each of the following , show whether or not the specified function is a solution to the corresponding differential equation.

(a) $y'''' + y''' + y' - y = 0$, $y(x) = \cos(x)$

(b) $\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + 6u\frac{\partial u}{\partial x} = 0$, $u(x,t) = \frac{1}{2}c\text{sech}^2\left[\frac{\sqrt{c}}{2}(x - ct - a)\right]$

(c) $y'' - y = 0$, $y(x) = \sinh(x)$

Problem 2 (Solving differential equations). For each of the following differential equations, do the following

(i) Identify the type of differential equation

(ii) Find the "general solution"

(a) $y' = 2y + 3$

(b) $y' = \frac{x^2 - y^2}{x+y}$

(c) $\sin(u)\frac{du}{dt} = \cos(u)/(1 + t^2)$

(d) $\frac{dy}{dt} = \frac{t^2 - y^2}{ty}$

(e) $(3x - 4y)dy = (2x + 7y)dx$

(f) $\frac{dy}{dt} + y/t = 6\cos(4t)$

(g) $y' + y = \cos(t)$

(h) $y' = 1 - y^3$

Problem 3 (Waaaaait a minute!). Explain what is wrong with the following argument:

Consider the differential equation

$$y' = 1 - 2y$$

Integrating both sides, we get the equation

$$y = y - y^2 + C.$$

Simplifying this, we get the solution $y^2 = C$ meaning that

$$y = \pm\sqrt{C}.$$

Problem 4 (Slope fields). For each of the following initial value problems

(i) Plot the slope field

(ii) Based on the plot of the slope field, predict the behavior of a solution to the IVP at large values of t

(iii) Explicilty solve the IVP

(iv) Based on the explicit solution of the IVP, determine the behavior at large values of t

(a) $y' = y(1 - y^2)$, $y(0) = 1$

(b) $y' = y(1 - y^2)$, $y(0) = 1/2$

(c) $y' = y(1 - y^2)$, $y(0) = 3/2$

Problem 5 (Second order equations). Consider the second order differential equation

$$y'' - y = 0$$

(a) Show that the change of variables $z = y' + y$ in the above second-order equation transforms it into the first order equation

$$z' - z = 0$$

(b) Find the general solution of the first-order equation of (a)

(c) By substituting the value of z back into the equation $z = y' + y$, find the value of y. Your final answer for y should involve *two* arbitrary constants.

Problem 6 (Solving Initial Value Problems). Find a solution to each of the following initial value problems

(a) $y' = x\cos(y)$, $y(0) = 1$

(b) $y' = e^x + y$, $y(1) = 2$

(c) $\frac{dy}{dt} + 2y = te^{-2t}$, $y(1) = 0$

(d) $xy' + 2y = \sin(x)$, $y(\pi/2) = 1$

Problem 7 (An almost homogeneous equation). Consider the differential equation

$$y' = x\cos(y/x) + y/x$$

(a) Explain why this is not a homogeneous differential equation

(b) Find the general solution of the differential equation.