

MATH 307: Problem Set #3

Due on: February 3, 2016

Problem 1 *Linear vs Nonlinear Equations*

In each of the following problems, determine **without solving the problem** an interval in which the solution of the initial problem is guaranteed to exist

(a) $y' + \tan(t)y = \sin(t)$, $y(\pi) = 0$

(b) $(4 - t^2)y' + 2ty = 3t^2$, $y(1) = -3$

.....

Problem 2 *Autonomous Equations*

Recall that an equilibrium solution of an autonomous equation is called *stable* if solutions lying on both sides of it tend toward it; is called *unstable* if solutions lying on both sides tend away from it; and is called *semistable* if solutions lying on one side of it tend toward it, while solutions on the other side tend away. In Prob 2.i-2.vi, please do each of the following

(a) Sketch a graph of $f(y)$ versus y

(b) Determine the critical (equilibrium) points

(c) Classify the critical points as stable, semistable, or unstable

(d) Draw the phase line and sketch several graphs of the solution in the ty -plane.

(i) $dy/dt = ay + by^2$, where $a > 0$, $b > 0$ and $y_0 \geq 0$

(ii) $dy/dt = ay + by^2$, where $a > 0$, $b > 0$ and $-\infty < y_0 < \infty$

(iii) $dy/dt = e^y - 1$, $-\infty < y_0 < \infty$

(iv) $dy/dt = e^{-y} - 1$, $-\infty < y_0 < \infty$

(v) $dy/dt = y(1 - y^2)$, $-\infty < y_0 < \infty$

(vi) $dy/dt = y^2(4 - y^2)$, $-\infty < y_0 < \infty$

.....

Problem 3 *Integrating Factors*

For each of the following, find an integrating factor and solve the given equation

(i) $y' = e^{2x} + y - 1$

(ii) $ydx + (2xy - e^{-2y})dy = 0$

(iii) $\left(3y + \frac{\sin(y)}{x^2y}\right) dx + \left(2x + \frac{\cos(y)}{xy}\right) dy = 0$ [Hint: Try $\mu(x, y) = x^a y^b$]

(iv) $\left(3x + \frac{6}{y}\right) + \left(\frac{x^2}{y} + 3\frac{y}{x}\right) \frac{dy}{dx} = 0$ [Hint: Try $\mu(x, y) = \mu(xy)$]

.....

Problem 4 *Euler's Method*

For Prob 1.i and 1.ii please do each of the following

(a) Find approximate values of the solution of the given value problem in the interval $[0, 0.5]$ with $\Delta t = 0.100$ using Euler's method. Record your results as a table of values in your writeup.

(b) Find approximate values of the solution of the given value problem in the interval $[0, 0.5]$ with $\Delta t = 0.050$ using Euler's method. Record your results as a table of values in your writeup.

(c) Find approximate values of the solution of the given value problem in the interval $[0, 0.5]$ with $\Delta t = 0.025$ using Euler's method. Record your results as a table of values in your writeup.

(d) Find the exact solution to the initial value problem.

(e) Compare your results in parts (a), (b), (c), and (d) by plotting them all in the same graph. Be sure that your plot is clear enough that one can tell which curves correspond to each part.

(i) $y' = 2y - 1, y(0) = 1$

(ii) $y' = 3 \cos(t) - 2y, y(0) = 0$

.....