

# MATH 307: Problem Set #6

Due on: Feb 26, 2016

## **Problem 1** *Trigonometric Forcing*

Find a particular solution to each of the following differential equations

- (a)  $y'' + 2y' + y = \sin(t)$
- (b)  $y'' + 2y' + y = \cos(t)$
- (c)  $y'' + 2y' + y = 3 \sin(t) + 2 \cos(t)$
- (d)  $y'' + y = \cos(t)$
- (e)  $y'' + y = e^{-2t} \sin(t)$
- (f)  $y'' + 2y' + 2y = e^{-t} \cos(t)$

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## **Problem 2** *Trigonometry Exercise*

In each of the following, determine  $\omega_0, R, \delta$  so as to write the given expression in the form  $u = R \cos(\omega_0 t - \Delta)$ .

- (a)  $u = 3 \cos(2t) + 4 \sin(2t)$
- (b)  $u = 4 \cos(3t) - 2 \sin(3t)$
- (c)  $u = -2 \cos(\pi t) - 3 \sin(\pi t)$

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## **Problem 3** *A Spring Problem*

A mass weighing 3 lbs stretches a spring 3 inches. If the mass is pushed upward, contracting the spring a distance of 1 in., and then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position  $u$  of the mass at any time  $t$ . Determine the frequency, period, amplitude, and phase of motion.

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**Problem 4** *Another Spring Problem*

A spring is stretched 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass is 5 m/s. If the mass is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm/s, determine its position  $u$  at any time  $t$ . Find the quasifrequency  $\mu$  and the ratio of  $\mu$  to the natural frequency corresponding to undamped motion.

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**Problem 5** *LCR Circuit Problem*

If a series circuit has a capacitor of  $C = 0.8 \times 10^{-6}$  F and an inductor of  $L = 0.2$  H, find the smallest value of the resistance  $R$  so that the circuit is critically damped.

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**Problem 6** *A Forced Spring Problem*

A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of  $10 \sin(t/2)$  N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s, formulate the initial value problem describing the motion of the mass.

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**Problem 7** *Another Forced Spring Problem*

If an undamped spring-mass system with a mass that weighs 6 lb and a spring constant of 41 lb/in is suddenly set in motion at  $t = 0$  by an external force of  $4 \cos(7t)$  lb, determine the position of the mass at any time and draw a graph of the displacement versus  $t$ .

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**Problem 8** *A Third Forced Spring Problem*

A mass that weighs 8 lb stretches a spring 6 inches. The system is acted on by an external force of  $8 \sin(8t)$  lb. If the mass is pulled down 3 in and then released, determine the position of the mass at any time. Determine the first four times at which the velocity of the mass is zero.

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**Problem 9** *Continuity Problem*

In each of the following sketch a graph of the function and determine whether it is continuous, piecewise continuous, or neither on the interval  $0 \leq t \leq 3$ .

(a)

$$f(t) = \begin{cases} t^2, & 0 \leq t \leq 1 \\ 1, & 1 < t \leq 2 \\ 3 - t, & 2 < t \leq 3 \end{cases}$$

(b)

$$f(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 3 - t, & 1 < t \leq 2 \\ 1, & 2 < t \leq 3 \end{cases}$$

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