# Math 307 Quiz 1

January 24, 2016

**Problem 1.** Give an example of each of the following

(a) A nonlinear, separable equation

(b) A fourth order, linear equation

(c) An exact equation

(d) A homogeneous equation

**Solution 1.**

(a) $y' = \sin(y)\cos(x)$.

(b) $y'''' = 0$.

(c) $y + xy' = 0$.

(d) $y' = (y/x) + 42$.

**Problem 2.** Find a family of solutions of the following differential equation

$$y' = y(1-y)(1+y)x.$$

**Solution 2.** This equation is separable. Separating and integrating as usual, we obtain

$$\int \frac{1}{y(1-y)(1+y)} dy = \int x dx.$$

The second integral gives us $\frac{1}{2}x^2 + C$. To do the first integral, we need to use partial fraction decomposition. Doing so, we obtain

$$\frac{1}{y(1-y)(1+y)} = \frac{1}{y} + \frac{-1/2}{y-1} + \frac{-1/2}{y+1}.$$

From this, we obtain (ignoring the constant of integration):

$$\begin{aligned}\int \frac{1}{y(1-y)(1+y)}dy &= \int \left(\frac{1}{y} - \frac{1/2}{y-1} - \frac{1/2}{y+1}\right) dy \\ &= \ln|y| - \frac{1}{2}\ln|y-1| - \frac{1}{2}|y+1| \\ &= \frac{1}{2}\ln\left|\frac{y^2}{y^2-1}\right|.\end{aligned}$$

The family of solutions we obtian is therefore:

$$\frac{1}{2}\ln\left|\frac{y^2}{y^2-1}\right| = \frac{1}{2}x^2 + C.$$

More simply, this says

$$\frac{y^2}{y^2-1} = Be^{x^2}.$$

In fact, we can solve for $y$, obtaining

$$y = \pm\sqrt{\frac{Be^{x^2}}{Be^{x^2}-1}}.$$

**Problem 3.** Find a particular solution of the initial value problem

$$y' = y - x,\ \ y(0) = 0.$$

**Solution 3.** This equation is linear, so we use an integrating factor. Recall that for an equation of the form

$$y' = p(x)y + q(x),$$

the integrating factor is given by

$$\mu(x) = \exp(-\int p(x)dx).$$

In our situation, $p(x) = 1$, and $q(x) = -x$, so we obtain $\mu(x) = e^{-x}$. The general solution is then given by

$$y = \frac{1}{\mu(x)}\int \mu(x)q(x)dx = e^x \int -xe^{-x}dx.$$

Now integration by parts tells us:

$$\int -xe^{-x}dx = xe^{-x} + e^{-x} + C.$$

Therefore we find the family of solutions

$$y = x + 1 + Ce^{x}.$$

Now since $y(0) = 0$, we see $0 = 0 + 1 + Ce^{0}$, and therefore $C = -1$. Thus the solution of the initial value problem is

$$y = x + 1 - e^{x}.$$