Math 307 Quiz 4

February 18, 2016

Problem 1. Find the general solution of each of the following differential equations

(a) $y'' + 6y' + 9y = 0$.

(b) $y'' + 2y' - 3y = 0$.

(c) $y'' + y' + 2y = 0$.

Solution 1.

(a) The characteristic polynomial is $r^2 + 6r + 9$, which has roots $-3, -3$. Therefore the general solution is

$$y = (At + B)e^{-3t}.$$

(b) The characteristic polynomial is $r^2 + 2r - 3$, which has roots $1, -3$. Therefore the general solution is

$$y = Ae^t + Be^{-3t}.$$

(c) The characteristic polynomial is $r^2 + r + 2$, which has roots $-\frac{1}{2} \pm i\frac{\sqrt{7}}{2}$. Therefore the general solution is

$$y = Ae^{-t/2}\cos(\sqrt{7}t/2) + Be^{-t/2}\sin(\sqrt{7}t/2).$$

Problem 2. Find the general solution of the differential equation

$$y'' + xy' - y = 0,$$

[Hint: $y_1 = x$ is a solution.]

Solution 2. We use the method of reduction of order. We propose a solution of the form $y = zy_1 = xz$ for some unknown function z. Then

$$y' = z + xz', \quad y'' = xz'' + 2z',$$

and plugging this into the original differential equation, we find

$$xz'' + 2z' + x^2z' = 0.$$

This becomes a first-order differential equation if we substitute $w = z'$:

$$xw' + (2 + x^2)w = 0.$$

This equation is separable! The solution is

$$w = Cx^{-2}e^{-x^2/2}.$$

This tells us that

$$z = C\int x^{-2}e^{-x^2/2}dx,$$

and therefore that

$$y = Cx\int x^{-2}e^{-x^2/2}dx.$$