Math 307 Quiz 1

January 21, 2015

Problem 1. Determine the general solution to the system of equations

$$\begin{aligned} x + 2y + 3z &= 0 \\ 4x + 5y + 6z &= 0 \\ 7x + 8y + 9z &= 0 \end{aligned}$$

Solution 1. We write down the corresponding augmented matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{bmatrix}.$$

The RREF form of this is

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Therefore we obtain the equivalent system of equations

$$\begin{aligned} x - z &= 0 \\ y + 2z &= 0 \end{aligned}$$

A general solution to the system equations is then given by

$$x = z, y = -2z, z = z$$

parametrized by the free variable z.

Problem 2. Consider the vectors

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \ \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Give an example of a vector in span$\{\vec{u}, \vec{v}\}$

(b) Give an example of a vector not in span$\{\vec{u}, \vec{v}\}$

Solution 2.

(a) There are lots of examples. Some are $\vec{0}$, $\vec{u}$ and $\vec{v}$.

(b) There are also lots of examples. One such example is

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Problem 3. Give an example of a matrix in EF but not in RREF.

Solution 3. There are lots of examples. One is

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Problem 4. Write down what it means for x_i to be a free variable of a linear system in echelon form.

Solution 4. The variable x_i is a free variable if it is NOT a leading variable of any of the equations in the linear system.