Math 307 Quiz 3

March 2, 2015

Problem 1. Define what it means for a function $f : \mathbb{R}^m \to \mathbb{R}^n$ to be a linear transformation.

Solution 1. The function f is linear if for all $\vec{u}, \vec{v} \in \mathbb{R}^m$ and all scalars c,

$$f(\vec{u} + \vec{v}) = f(\vec{u}) + f(\vec{v})$$

and also

$$f(c\vec{u}) = cf(\vec{u}).$$

Problem 2. Define the range of a function $f : \mathbb{R}^m \to \mathbb{R}^n$.

Solution 2. The range of f is the set

$$\text{Range}(f) = \{f(\vec{v}) : \vec{v} \in \mathbb{R}^m\}.$$

Problem 3. Define what it means for a function $f : \mathbb{R}^m \to \mathbb{R}^n$ to be onto.

Solution 3. The function f is onto if every $\vec{w} \in \mathbb{R}^n$ is mapped to by at least one vector $\vec{v} \in \mathbb{R}^m$.

Problem 4. Give an example of a function $f : \mathbb{R}^2 \to \mathbb{R}^3$ that is not linear.

Solution 4. There are lots of examples! One such example is

$$f\left(\left[\begin{array}{c} x, y \end{array}\right]\right) = \left[\begin{array}{c} x+1 \\ x \\ y \end{array}\right].$$

Problem 5. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be the function defined by $f(\vec{x}) = A\vec{x}$ for

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Prove that f is onto.

Solution 5. The row reduced echelon form of A is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Therefore the column vectors of A are linearly independent. Hence $f(x)$ is one-to-one. Since $f(x)$ is a map from $\mathbb{R}^3$ to $\mathbb{R}^3$, this also implies that f is onto.