Math 307 Quiz 4

March 4, 2015

Problem 1. Let $V \subseteq \mathbb{R}^n$ be a subset. Define what it means for V to be a subspace of $\mathbb{R}^n$.

Solution 1. The subset V is a subspace of $\mathbb{R}^n$ if it satisfies the following three properties:

(a) V is nonempty

(b) V is closed under vector addition, ie. for all $\vec{u}, \vec{v} \in V$, the vector $\vec{u}+\vec{v} \in V$

(c) V is closed under scalar multiplication, ie. for all $\vec{v} \in V$ and scalars c, the vector $c\vec{v} \in V$

Problem 2. Give an example of a subspace V of $\mathbb{R}^2$ that has more than one element, but that is not all of $\mathbb{R}^2$

Solution 2. There are lots of examples! A very basic one is to simply take the span of a nonzero vector, ie.

$$V = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

Problem 3. Show that

$$V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 2y - z = 0 \right\}$$

is a subspace of $\mathbb{R}^3$.

Solution 3. We can do very easily by noting that the linear function $f : \mathbb{R}^3 \to \mathbb{R}$ defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x + 2y - z$$

has kernel equal to V:

$$\ker(f) = V.$$

We proved in class that the kernel of a linear function is a subspace, so this proves that V is a subspace.

As an alternative solution, one can check that V satisfies the definition of a subspace.

Problem 4. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Determine the inverse of A.

Solution 4. To find the inverse of A, we make the augmented matrix

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right],$$

and row reduce it, arriving at the matrix

$$[I|A^{-1}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right].$$

Therefore

$$A^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \end{bmatrix}.$$