MATH 309: Homework #3

Due on: November 9, 2015

Problem 1 *Boundary Value Problems*

For each of the following boundary value problems, find all solutions to the boundary value problem or show that no solution exists.

(a) $y'' + y = 0$, $y(0) = 0$, $y'(\pi) = 1$

(b) $y'' + y = 0$, $y(0) = 0$, $y(L) = 0$

(c) $y'' + y = x$, $y(0) = 0$, $y(\pi) = 0$

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Problem 2 *Dirichlet Eigenvalue Problem*

Determine for which values of λ the boundary value problem

$$y'' + \lambda y = 0,\ y(0) = 0,\ y(L) = 0,$$

has a solution and describe the solutions.

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Problem 3 *von Neumann Eigenvalue Problem*

Determine for which values of λ the boundary value problem

$$y'' + \lambda y = 0,\ y'(0) = 0,\ y'(L) = 0,$$

has a solution and describe the solutions.

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Problem 4 *Fourier Series*

For each of the following functions, sketch a graph of the function and find the Fourier series

(a) $f(x) = \sin^3(x) + \cos^2(2x+3)$

(b) $f(x) = -x, \ -L \leq x < L$ with $f(x+2L) = f(x)$ for all x

(c) $f(x) = \begin{cases} x+1, & -\pi \leq x < 0 \\ 1-x, & 0 \leq x < \pi \end{cases}$ with $f(x+2\pi) = f(x)$ for all x

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Problem 5 *Parseval's Identity*

Let $f(x)$ be a periodic function with fundamental period $2L$, and suppose that

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right]$$

Using the fact that

$$\left\{ \frac{1}{2}, \cos\left(\frac{n\pi x}{L}\right), \sin\left(\frac{m\pi x}{L}\right) : n = 0, 1, 2, \ldots, m = 1, 2, 3, \ldots \right\}$$

is a mutually orthogonal set of functions, prove Parseval's identity:

$$\frac{1}{L} \int_{-L}^{L} f(x)^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

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Problem 6 *Parseval's Identity Application*

Use Parseval's identity and the Fourier series for the square wave function

$$f(x) = \begin{cases} 0, & -1 \leq x < 0 \\ 1, & 0 \leq x < 1 \end{cases}, \text{ with } f(x+2) = f(x) \text{ for all } x$$

to obtain the value of the infinite sum

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

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