Math 309 Quiz 8

December 11, 2015

Problem 1. Find a solution to the wave equation problem

$$u_{tt} - c^2 u_{xx} = 0$$

$$u(0,t) = 0, \quad u(2\pi, t) = 0$$

$$u(x,0) = \sin(3x/2) - 2\sin(7x), \; u_t(x,0) = 0.$$

Solution 1. Note that $L = 2\pi$ and therefore the solution is of the form

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin(nx/2)\cos(cnt/2)$$

for some constants b_n. Therefore

$$u(x,0) = \sum_{n=1}^{\infty} b_n \sin(nx/2),$$

and since $u(x,0) = \sin(3x/2) - 2\sin(7x)$ it is clear from inspection (without integrating!!!) that $b_3 = 1$, $b_{14} = -2$ and $b_n = 0$ otherwise. Therefore

$$u(x,t) = \sin(3x/2)\cos(c3t/2) - 2\sin(7x)\cos(c7t).$$

Problem 2. Find a solution to the wave equation problem

$$u_{tt} - c^2 u_{xx} = 0$$

$$u(0,t) = 0, \quad u(\pi, t) = 0$$

$$u(x,0) = 0, \; u_t(x,0) = \sin(3x) - 7\sin(5x).$$

Solution 2. Note that $L = \pi$ and therefore the solution is of the form

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin(nx)\sin(cnt)$$

(note the difference from the previous problem!!!) for some constants b_n. Therefore

$$u_t(x,t) = \sum_{n=1}^{\infty} cnb_n \sin(nx)\cos(cnt),$$

so that

$$u_t(x,0) = \sum_{n=1}^{\infty} cnb_n \sin(nx),$$

and since $u_t(x,0) = \sin(3x) - 7\sin(5x)$ it is clear from inspection (without integrating!!!) that $c3b_3 = 1$, $c5b_5 = -7$ and $b_n = 0$ otherwise. Therefore

$$u(x,t) = \frac{1}{3c}\sin(3x)\sin(c3t) - \frac{7}{5c}\sin(5x)\sin(c5t).$$