Math 309 Section F **Name (Print):** ____________________
Fall 2015
Final **Student ID:** ____________________
December 17, 2015
Time Limit: 1 Hour 50 Minutes

This exam contains 11 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a single, handwritten, one-sided notesheet and a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	15	
4	20	
5	20	
6	10	
7	15	
8	10	
9	10	
Total:	120	

1. (10 points) Find the general solution of the equation

$$\frac{d}{dt}\vec{y}(t) = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \vec{y}(t) + \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t.$$

2. (a) (5 points) Find the solution of the heat equation problem

$$u_t - 3u_{xx} = 0, \quad u(0,t) = 0, \; u(4,t) = 0, \quad u(x,0) = \sin(\pi x) - 2\sin(3\pi x/4).$$

(b) (5 points) Find the solution of the heat equation problem

$$u_t - 5u_{xx} = 0, \quad u_x(0,t) = 0, \; u_x(6,t) = 0, \quad u(x,0) = 4 - 2\cos(\pi x/3) + 7\cos(3\pi x/2).$$

3. (15 points) Consider the wave equation problem

$$u_{tt} - c^2 u_{xx} = 0$$

$$u(0,t) = 0, u(L,t) = 0, \ t > 0$$

$$u(x,0) = f(x), u_t(x,0) = 0, \ 0 \leq x \leq 7$$

where $L = 7$, $c = 1$, and

$$f(x) = \begin{cases} 0, & 0 \leq x < 3 \\ 1, & 3 \leq x < 4 \\ 0, & 4 \leq x < 7 \end{cases}$$

(a) Find the Fourier series solution $u(x,t)$ of the above wave equation for $0 \leq x \leq 7$ and $t > 0$ (eg. do not use D'Alembert)

(b) Sketch a graph of the odd, $2L$-periodic extension of $f(x)$, including at least two full periods

(c) Sketch a graph of the solution $u(x,t)$ when $t = 4$ (for this you should use D'Alembert).

4. (20 points) Find the solution of Laplace's equation

$$u_{xx} + u_{yy} = 0,$$

inside the interior of the rectangle bounded by the lines $x = 0, x = 1, y = 0$, and $y = 2$, and satisfying the boundary conditions (for $0 \leq x \leq 1$ and $0 \leq y \leq 2$):

$$u(x, 0) = 0,\ u(x, 2) = 0,\ u(0, y) = 0,\ u(1, y) = 1 - |y - 1|.$$

5. (20 points) Find the general solution of the system of equations

$$\frac{d}{dt}\vec{y}(t) = A\vec{y}(t), \quad A = \begin{pmatrix} 3 & -2 & 2 \\ -6 & 7 & 2 \\ -6 & 6 & 3 \end{pmatrix}.$$

6. (10 points) Use separation of variables to convert the PDE

$$u_{xx} + u_{xt} + u_t = 0$$

into two second-order ODEs.

7. (15 points) Calculate the Fourier series of the function

$$f(x) = x^2 - x, \ -1 \leq x \leq 1$$

with $f(x+2) = f(x)$ for all x.

8. (10 points) Find the solution of the heat equation problem

$$u_t - 3u_{xx} = 0, \quad u(0,t) = 3, \; u(1,t) = 1, \quad u(x,0) = -3(x^2 - 1) + 3x$$

9. (10 points) Find a solution of the wave equation problem

$$u_{tt} - c^2 u_{xx} = 0$$

$$u(0,t) = 0, u(L,t) = 0,\ t > 0$$

$$u(x,0) = 0,\ u_t(x,0) = g(x),\ 0 \leq x \leq 1$$

where $L = 1$, $c = 2$, and $g(x) = x$ for $0 \leq x \leq 1$.