MATH 309: Homework #5

Due on: May 18, 2015

Problem 1   *Boundary Value Problems*

For each of the following boundary value problems, find all solutions to the boundary value problem or show that no solution exists.

(a) $y'' + y = 0$, $y(0) = 0$, $y'(\pi) = 1$

(b) $y'' + y = 0$, $y(0) = 0$, $y(L) = 0$

(c) $y'' + y = x$, $y(0) = 0$, $y(\pi) = 0$

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Problem 2   *Dirichlet Eigenvalue Problem*

Determine for which values of λ the boundary value problem

$$y'' + \lambda y = 0, \ \ y(0) = 0, \ \ y(L) = 0,$$

has a solution and describe the solutions.

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Problem 3   *Neumann Eigenvalue Problem*

Determine for which values of λ the boundary value problem

$$y'' + \lambda y = 0, \ \ y'(0) = 0, \ \ y'(L) = 0,$$

has a solution and describe the solutions.

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Problem 4   *Even and Odd Functions*

Prove that any function $f(x)$ may be expressed as a sum of two functions $f(x) = g(x) + h(x)$ with $g(x)$ even and $h(x)$ odd. [Hint: consider $f(x) + f(-x)$].

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Problem 5   *Even and Odd Functions*

Prove that the derivative of an even function is odd and that the derivative of an odd function is even.

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Problem 6   *Sine Series*

Consider the function
$$f(x) = \begin{cases} 0, & 0 < x < \pi \\ 1, & \pi < x < 2\pi \\ 2, & 2\pi < x < 3\pi \end{cases}$$

(a) Scketch a graph of $f(x)$

(b) By reflecting f appropriately, express f as a sine series.

(c) Plot three different partial sums of the sine series, clearly indicating the partial sums being plotted.

(d) Sketch a graph of the function to which the sine series converges for three periods.

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Problem 7   *Cosine Series*

Consider the function
$$f(x) = \begin{cases} x, 0 < x < \pi \\ 0, \pi < x < 2\pi \end{cases}$$

(a) Scketch a graph of $f(x)$

(b) By reflecting f appropriately, express f as a cosine series.

(c) Plot three different partial sums of the cosine series, clearly indicating the partial sums being plotted.

(d) Sketch a graph of the function to which the cosine series converges for three periods.

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