

# Math 309 Quiz 5 (Groups)

May 25, 2016

**Problem 1.** Consider the wave problem

$$\begin{aligned}u_{tt} &= c^2 u_{xx} \\u(0, t) &= 0, \quad u(L, t) = 0 \\u(x, 0) &= 0, \quad u_t(x, 0) = g(x)\end{aligned}$$

where here

$$g(x) = \begin{cases} 4x/L, & 0 \leq x < L/4 \\ 1, & L/4 \leq x < 3L/4 \\ 4(L-x)/L & 3L/4 \leq x \leq L \end{cases}$$

(a) Find a solution to the wave equation of the form

$$u(x, t) = \sum_{n=1}^{\infty} a_n \sin(n\pi ct/L) \sin(n\pi x/L)$$

for some constants  $a_n$

(b) If the vibration of a string is described by  $u(x, t)$  as in (a) then the energy in frequency  $\omega_n = n/(2L)$  is given by

$$E_n = \frac{1}{2} K c^2 n a_n^2$$

where  $K$  is some constant having to do with the material properties of the string. Use your answer in (a) to plot the energy  $E_n$  as a function of the frequency  $f_n$  (take  $K = 1, c = 1$ ). In which frequency is the energy largest? What happens to the energy as  $n \rightarrow \infty$ ?

**Problem 2.** Consider the same wave problem as in Problem 1

(a) Use the method of d'Alembert to find a solution of the wave problem in Problem 1, with  $c = 4$  and  $L = 1$ .

(b) Plot  $u(x, 1/8)$ .