MATH 309: Homework #2

Due on: April 17, 2017

Problem 1 *Jordan Normal Form*

For each of the following values of the matrix A, find an invertible matrix P and a matrix N in Jordan normal form such that $P^{-1}AP = N$.

(a)

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

(d)

$$A = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$$

(e)

$$A = \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}$$

(f)

$$A = \begin{pmatrix} 0 & 0 & 24 \\ 1 & 0 & 2 \\ 0 & 1 & -5 \end{pmatrix}$$

(g)

$$A = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & -3 \\ 0 & 1 & -3 \end{pmatrix}$$

(h)

$$A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix}$$

(i)

$$A = \begin{pmatrix} -1 & -1 & 0 \\ 4 & 3 & 0 \\ -6 & -3 & 1 \end{pmatrix}$$

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Problem 2 **Matrix Exponential**

For each of the values of the matrix A in the previous problem, determine the value of $\exp(At)$

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Problem 3 Fundamental Matrix

Find a fundamental matrix for each of the following systems of equations

(a)

$$\begin{aligned} x' &= x + y \\ y' &= x - y \end{aligned}$$

(b)

$$\begin{aligned} x' &= -x - 4y \\ y' &= x - y \end{aligned}$$

(c)

$$\begin{aligned} x' &= x + y \\ y' &= 4x - 2y \end{aligned}$$

(d)

$$\begin{aligned} x' &= -x - 4y \\ y' &= x - y \end{aligned}$$

(e)

$$\begin{aligned} x' &= x - y \\ y' &= 5x - 3y \end{aligned}$$

(f)

$$\begin{aligned} x' &= 3x - 4y \\ y' &= x - y \end{aligned}$$

(g)

$$\begin{aligned} x' &= 4x - 8y \\ y' &= 8x - 4y \end{aligned}$$

. . . .

Problem 4 Uniqueness of Fundamental Matrix

Let $A(t)$ be a matrix continuous on the interval (α, β). Show that if $\Psi(t)$ and $\Phi(t)$ are two fundamental matrices for the equation

$$\vec{y}'(t) = A(t)\vec{y}(t)$$

on the interval (α, β), then there exists a (constant) invertible matrix P so that $\Phi(t) = \Psi(t)P$.

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