

# MATH 309: Homework #5

Due on: May 26, 2017

## **Problem 1** *Boundary Value Problems*

For each of the following boundary value problems, find all solutions to the boundary value problem or show that no solution exists.

(a)  $y'' + y = 0, y(0) = 0, y'(\pi) = 1$

(b)  $y'' + y = 0, y(0) = 0, y(L) = 0$

(c)  $y'' + y = x, y(0) = 0, y(\pi) = 0$

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## **Problem 2** *Dirichlet Eigenvalue Problem*

Determine for which values of  $\lambda$  the boundary value problem

$$y'' + \lambda y = 0, y(0) = 0, y(L) = 0,$$

has a solution and describe the solutions.

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## **Problem 3** *Neumann Eigenvalue Problem*

Determine for which values of  $\lambda$  the boundary value problem

$$y'' + \lambda y = 0, y'(0) = 0, y'(L) = 0,$$

has a solution and describe the solutions.

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**Problem 4** *Even and Odd Functions*

Prove that any function  $f(x)$  may be expressed as a sum of two functions  $f(x) = g(x) + h(x)$  with  $g(x)$  even and  $h(x)$  odd. [Hint: consider  $f(x) + f(-x)$ ].

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**Problem 5** *Even and Odd Functions*

Prove that the derivative of an even function is odd and that the derivative of an odd function is even.

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**Problem 6** *Sine Series*

Consider the function

$$f(x) = \begin{cases} 0, & 0 < x < \pi \\ 1, & \pi < x < 2\pi \\ 2, & 2\pi < x < 3\pi \end{cases}$$

- (a) Sketch a graph of  $f(x)$
- (b) By reflecting  $f$  appropriately, express  $f$  as a sine series.
- (c) Plot three different partial sums of the sine series, clearly indicating the partial sums being plotted.
- (d) Sketch a graph of the function to which the sine series converges for three periods.

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**Problem 7** *Cosine Series*

Consider the function

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$

- (a) Sketch a graph of  $f(x)$
- (b) By reflecting  $f$  appropriately, express  $f$  as a cosine series.
- (c) Plot three different partial sums of the cosine series, clearly indicating the partial sums being plotted.
- (d) Sketch a graph of the function to which the cosine series converges for three periods.

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