Math 309 Quiz 1 Practice

April 5, 2017

Problem 1. For each of the following, write TRUE if the statement is true and BANANAS if the statement is false. Throughout A is an $n \times n$ matrix.

(a) The geometric multiplicity of an eigenvalue λ of A is always larger than the algebraic multiplicity.

(b) The sum of the algebraic multiplicities of all of the eigenvalues of A is n.

(c) The geometric multiplicity of an eigenvalue λ of A is equal to the nullity of $A - \lambda I$.

(d) A square matrix is singular if and only if its determinant is nonzero.

Problem 2. For the given matrix A, determine the following

- The eigenvalues of A

- The geometric and algebraic multiplicity of each eigenvalue of A

- A basis for the eigenspace of each eigenvalue

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Problem 3. Give an example of a 2×2 matrix A which is degenerate (ie. has an eigenvalue whose algebraic and geometric multiplicities do not agree).