Math 309 Quiz 1 Solutions

April 13, 2017

Problem 1. For each of the following, write TRUE if the statement is true and BANANAS if the statement is false. Throughout A is an $n \times n$ matrix.

(a) If A is an upper triangular matrix ,then the entries on the main diagonal of A are exactly the eigenvales of A.

(b) If the sum of the geometric multiplicities of all of the eigenvalues of A is n, then A is non-degenerate

(c) Eigenvalues cannot be zero

(d) The zero vector is never an eigenvector

Solution 1.

(a) True

(b) True

(c) False

(d) True

Problem 2. For the given matrix A fill in the following table:

λ	$m_a(\lambda)$	$m_g(\lambda)$	$E_\lambda(A)$ basis

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}.$$

Solution 2.

λ	$m_a(\lambda)$	$m_g(\lambda)$	$E_\lambda(A)$ basis
5	1	1	$\left\{\binom{1}{1}\right\}$
2	1	1	$\left\{\binom{-2}{1}\right\}$

Problem 3. Give an example of a 3×3 matrix A which is degenerate (ie. has an eigenvalue whose algebraic and geometric multiplicities do not agree).

Solution 3. Several examples include

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 4 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 2 & 3 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$