Math 309 Quiz 2 Solutions

May 30, 2017

Problem 1. TRUE or BANANAS? If the Jordan normal form of a matrix contains a Jordan block of size 2×2 or larger, then the matrix is degenerate.

Solution 1. TRUE

Problem 2. TRUE or BANANAS? Every eigenvector is also a generalized eigenvector.

Solution 2. TRUE

Problem 3. Give an example of a 3×3 degenerate matrix with two distinct eigenvalues. [Hint: pick a matrix in Jordan form]

Solution 3. One could take for example

$$\begin{pmatrix} 5 & 1 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 42 \end{pmatrix}$$

Problem 4. Write down the definition of a generalized eigenvector.

Solution 4. A generalized eigenvector of A with eigenvalue λ is a nonzero vector $\vec{v}$ satisfying the property that $(A-\lambda I)^m \vec{v} = \vec{0}$ for some integer $m > 0$. The degree of a generalized eigenvector is the smallest such value of m for which this holds.

Problem 5. Find a basis for the solution space of the system of differential equations

$$y_1' = 5y_1 + 2y_2$$
$$y_2' = 3y_1 + 4y_2$$

Solution 5. The eigenvalues of this matrix are 7 and 2, and the associated eigenspaces are

$$E_7(A) = \text{span}\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$$
$$E_2(A) = \text{span}\left\{\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right\}$$

This gives us a fundamental set of solutions

$$\left\{\begin{pmatrix} e^{7t} \\ e^{7t} \end{pmatrix}, \begin{pmatrix} -2e^{2t} \\ 3e^{7t} \end{pmatrix}\right\}$$