Math 309 Quiz 3 Practice

April 27, 2017

Problem 1. TRUE or BANANAS? The matrix exponential $\exp(A(t)t)$ is a fundamental matrix for the system of equations

$$\frac{d}{dt}\vec{y}(t) = A(t)\vec{y}(t).$$

Problem 2. TRUE or BANANAS? If $\Psi(t)$ and $\Phi(t)$ are two fundamental matrices for the system of equations

$$\frac{d}{dt}\vec{y}(t) = A(t)\vec{y}(t),$$

then there exists an invertible constant matrix C such that $\Psi(t) = \Phi(t)C$.

Problem 3. Suppose that A is a 2×2 matrix with complex eigenvalues $a \pm ib$ (with $b \neq 0$). Determine for which values of a and b the critical point at the origin of the system

$$\frac{d}{dt}\vec{y}(t) = A\vec{y}(t)$$

is spirally stable or spirally unstable.

Problem 4. Find a real basis for the solution space of the system of differential equations

$$y_1' = 2y_1 + y_2$$
$$y_2' = -y_1 + 2y_2$$