

Math 309 Quiz 4 Practice

May 18, 2017

Throughout these questions $L = T/2$ and \mathcal{P}_T will denote the set

$$\mathcal{P}_T = \left\{ f(x) \mid f(x+T) = f(x) \text{ for all } x \text{ and } \int_{-L}^L f(x)^2 dx < \infty \right\}.$$

Then

Problem 1. TRUE or BANANAS? Suppose $f(x)$ is in \mathcal{P}_T and has the Fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{m=1}^{\infty} b_m \sin\left(\frac{m\pi x}{L}\right).$$

Then a_0 is the average value of $f(x)$ on $[-L, L]$.

Problem 2. TRUE or BANANAS? Consider the periodic function with period 3 defined by

$$f(x) = \begin{cases} x & 0 \leq x < 1 \\ 1 & 1 \leq x < 2 \\ 0 & 2 \leq x < 3 \end{cases} \quad \text{with } f(x+3) = f(x) \text{ for all } x.$$

The Fourier series of $f(x)$ converges to 1 at $x = 1$.

Problem 3. TRUE or BANANAS? Let $f(x)$ be the same function as in the previous problem. The Fourier series of $f(x)$ converges to 1 at $x = 2$.

Problem 4. Calculate the Fourier series for the function

$$f(x) = x \text{ for } -L \leq x < L \text{ with } f(x+2L) = f(x) \text{ for all } x.$$