Math 309 Quiz 4 Practice Solutions

May 18, 2017

Throughout these questions $L = T/2$ and $\mathcal{P}_T$ will denote the set

$$\mathcal{P}_T = \left\{ f(x) | f(x+T) = f(x) \text{ for all } x \text{ and } \int_{-L}^{L} f(x)^2 dx < \infty \right\}.$$

Then

Problem 1. TRUE or BANANAS? Suppose $f(x)$ is in $\mathcal{P}_T$ and has the Fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{m=1}^{\infty} b_m \sin\left(\frac{m\pi x}{L}\right).$$

Then a_0 is the average value of $f(x)$ on $[-L, L]$.

Solution 1. TRUE.

Problem 2. TRUE or BANANAS? Consider the periodic function with period 3 defined by

$$f(x) = \begin{cases} x & 0 \leq x < 1 \\ 1 & 1 \leq x < 2 \\ 0 & 2 \leq x < 3 \end{cases} \qquad \text{with } f(x+3) = f(x) \text{ for all } x.$$

The Fourier series of $f(x)$ converges to 1 at $x = 1$.

Solution 2. TRUE (converges to 1 by the Pointwise Convergence Theorem).

Problem 3. TRUE or BANANAS? Let $f(x)$ be the same function as in the previous problem. The Fourier series of $f(x)$ converges to 1 at $x = 2$.

Solution 3. BANANAS (converges to 1/2 by the Pointwise Convergence Theorem).

Problem 4. Calculate the Fourier series for the function

$$f(x) = x \;\text{ for } -L \leq x < L \text{ with } f(x+2L) = f(x) \text{ for all } x.$$

Solution 4. The function $f(x)$ is odd, so we know immediately that $a_n = 0$ for all $n \geq 0$. Therefore we need only calculate the b_n's. To do so, we will use the Euler-Fourier formula:

$$\begin{aligned}
b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{m\pi x}{L}\right)dx \\
&= \frac{1}{L}\int_{-L}^{L} x\sin\left(\frac{m\pi x}{L}\right)dx \\
&= \frac{-1}{m\pi}x\cos\left(\frac{m\pi x}{L}\right)|_{-L}^{L} + \frac{1}{m\pi}\int_{-L}^{L}\cos\left(\frac{m\pi x}{L}\right)dx \\
&= \frac{-2L\cos(m\pi)}{m\pi} \\
&= \frac{2L}{m\pi}(-1)^{m+1}.
\end{aligned}$$