Math 309 Quiz 5 Solutions

May 30, 2017

Problem 1. Determine the cosine series of the 4π-periodic function $f(x) = \cos(x) + \cos(5x) - \cos(x/2)$ on the interval $[0, 2\pi]$.

Solution 1. We want to write

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

for some constants $a_0, a_1, \ldots$. Since the interval is $[0, 2\pi]$ the value of L is also 2π. Therefore

$$\cos(x) + \cos(5x) - \cos(x/2) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{nx}{2}\right)$$

To get this right, we just take $a_1 = -1, a_2 = 1, a_{10} = 1$ and $a_n = 0$ otherwise!

Problem 2. TRUE or BANANAS: The homogeneous Dirichlet boundary value problem

$$y'' + \lambda y = 0$$
$$y(0) = y_0 \quad y(L) = y_1$$

will have either no solutions, one solution, or infinitely many solutions.

Solution 2. TRUE: The boundary conditions lead to a (nonhomogeneous) linear system of two equations with two unknowns. This has either no solutions, one solution, or infinitely many solutions.

Problem 3. Calculate the sine series of $f(x) = x$ on the interval $[0, 1]$.

Solution 3. The function $f(x) = x$ is already odd, so we can just extend it to be it's usual self from $[-1, 0]$ and then perodically so that $f(x+2) = f(x)$ for all x. Then the Euler-Fourier formula says

$$f(x) = \sum_{m=1}^{\infty} a_m \sin(m\pi x)$$

for

$$\begin{aligned} a_m &= \frac{1}{1}\int_{-1}^{1} f(x)\sin(m\pi x)dx = 2\int_0^1 x\sin(m\pi x)dx \\ &= \frac{-2}{m\pi} x\cos(m\pi x)|_0^1 + \frac{2}{m\pi}\int_0^1 \cos(m\pi x)dx = (-1)^{m+1}\frac{2}{m\pi} \end{aligned}$$